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Application of differential Game Theory for vehicle collision avoidance

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Author’s Declaration

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This thesis is dedicated to my family for their support and encouragement.

Abstract

With respect to autonomous driving, recent advances in the innovative application of robotics have highlighted the importance of interaction between different actors, as robots are no longer confined to specific environments such as factory floors, but face a complex world with many other actors. Therefore, rational interaction between different actors is required to avoid conflicts. Autonomous actors must take into account the decisions of other actors when making decisions themselves (Zanardi et al., 2021c). In the present work, the approach adopted is to use discrete control functions instead of continuous control functions as generally used in differential games and in approaches of Zanardi et al. (2021c) and Mylvaganam et al. (2017). The reason for this is that in a differential game with two or more players, the computation of Nash Equilibria is time consuming and complex calculations have to be performed (Mylvaganam et al., 2017; Başar, 1986). For the different discrete control variables of the discrete control functions, the reachability analysis was used to define the different reachable states of the player, which also allows uncertainties for the initial state and the strategies of the players to be taken into account. Based on the reachable states, the cost for each player could be estimated and it is possible through the discrete control functions to consider and solve games in normal form instead of the complex and time-consuming solution process of differential games. Based on the defined reachable states, the individual costs for each player were estimated. The reachability analysis also allows to take into account the uncertainties in the initial state and in the strategy of the players. In order to be able to determine the costs for each player, two groups of cost functions were defined in this work, one containing cost functions that are intended to prevent collisions and the other containing running costs that take into account the speed and comfort of the occupant. In addition, the worst cases were estimated for each pair of controls included in the reachable states. Depending on the cost function, the cost functions were optimized with respect to the maximum or minimum. As part of the nonlinear optimization of the cost functions, optimization functions were defined in which the respective cost functions were adjusted with respect to the worst case. These defined cost functions were simulated using three defined driving scenarios, which include different driving maneuvers and evaluated with respect to their relevance. The simulation results have generally shown that the defined cost functions are reasonable and comprehensible cost functions that can be used in games with two players. A cost function that calculates costs related to collisions energy in the event of a possible collision did not provide the expected results due to the consideration of points as states. Since an exact intersection of points is unlikely due to different parameters, complexity and defined uncertainties, this is reflected in the simulation results. Further work could address this problem and consider other geometric bodies instead of points, so that comprehensible results can be simulated.

Acronyms

CORA	COntinuous Reachability Analyzer
CSI	Cervical Spine Injury
KS	Kinematic Single-track
NE	Nash Equilibrium
ODD	Operational Design Domain
OWT	One Way Traffic
PEGASUS	Project for the Establishment of Generally Accepted quality criteria, tools and methods as well as Scenarios and Situations
SAE	Society of Automotive Engineers
SSM	Surrogate Safety Measures
TTC	Time-To-Collision
TWT	Two Way Traffic
UDG	Urban Driving Games

Contents

1. Introduction

In order to make the coexistence of different road users safe, conflict-free and efficient, corresponding laws have been passed and guidelines have been drafted. Already in the first paragraph, section 1 of one of these regulations, the Road Traffic Regulations, it is pointed out that those who participate in road traffic should exercise uninterrupted caution as well as mutual consideration (StVO, 2019). Failure to adapt in this case can lead to critical situations, which can cause a traffic conflict (Lehsing, 2019). According to Kühnen (2000), a conflict is defined as two elements that want to perform incompatible or opposing actions at the same time. Here, driving simulation provides a method to explore and analyze traffic conflicts and driving behaviors that may be the reason for these conflicts (Fisher et al., 2011). The occurrence of conflicts depends on many parameters and influencing factors, which can also influence the severity of accidents. The large number of parameters and influencing factors ensures the high complexity of the conflict, which means that an exact prediction for the individually occurring conflicts requires a high effort and can only be generalized under assumptions.

With respect to autonomous driving, recent advances in the innovative application of robotics have highlighted the importance of interaction between actors, as robots are no longer merely located in defined environments, such as factory floors, but are confronted with a complex world with many other actors. Therefore, in order to avoid conflicts, rational interaction between actors is necessary (Jafary et al., 2018). It is also necessary for autonomous actors to consider the decisions of other actors when making decisions. For example, an autonomous vehicle should first consider the decisions and actions of other road users before the autonomous vehicle itself plans its next movements and actions based on them (Zanardi et al., 2021c). Initial work by Liniger and Lygeros (2019), Spica et al. (2020), Wang et al. (2021) and Williams et al. (2018) has already investigated autonomous racing scenarios where the actors' only goal is to cross the finish line first. In the context of this work, unlike racing scenarios, which are inherently counterproductive because they lead to a zero-sum problem, we consider actors on everyday roads. The interactions of the actors are decoupled from any constraints, in contrast to the racing scenarios.

In contrast to Zanardi et al. (2021c) and Mylvaganam et al. (2017), the education of Nash Equilibria in differential games is not considered in the context of this work, since in a differential game with two or more players the computation of the Nash Equilibira is time consuming and complex computations have to be performed, especially for nonlinear games (Mylvaganam et al., 2017; Başar, 1986). Therefore, discrete control functions are used in this work instead of continuous control functions as they are generally used in differential games. For the different discrete control variables of the discrete control functions, reachability analysis is used to define the different reachable states of the player. Reacha- bility analysis allows uncertainty to be taken into account. Based on the reachable states, the cost for each player can be estimated. Here, a state is not represented by a single vector, but by a geometric object. The reason for using geometric objects instead of vectors is that uncertainties in the state can be taken into account. With the Continuous Reachability Analyzer, one can basically take an initial set represented by a geometric object, a system dynamics, and a time horizon, and calculate all the states that are reachable from the initial state within the time horizon, given the system dynamics. Based on the discrete control functions, it is possible to consider and solve games in normal form rather than the complex and time-consuming solution process of differential games.

In Chapter 2, the theoretical foundations and the areas of Game Theory relevant to this work are explained. This includes, among others, the distinction between static games in Chapter 2.1.1 and extensive form games in Chapter 2.1.2 as well as the explanation of general-sum in Chapter 2.1.4 and zero-sum games in Chapter 2.1.3, cooperative and non-cooperative games in Chapter 2.1.5 and the explanation of differential games as part of dynamic games in Chapter 2.2. In Chapter 3 the current state of the art is explained, followed by the Methodological approach in Chapter 4. In Chapter the normal form game is created based on the methodological approach. In addition to the structure of the normal form game in Chapter 5.1, the scenarios are modeled in Chapter 5.2 before the cost functions for the normal form game are defined in the following Chapters 5.5, 5.3 and 5.5. The results for the agents of the game resulting from the normal form game are evaluated in Chapter 6 for the cost functions and in Chapter 7. Finally, the Chapter 8 gives an overview of the obtained knowledge as well as results of this work and gives an outlook on what future work could deal with.

2. Preliminaries

This Chapter serves as a theoretical foundation and understanding of this work. The Chapter 2.1 gives a general introduction to Game Theory, its different strategy possibilities and provides a general definition of a game. Furthermore, the Chapter 2.1.1 characterizes the properties of a static form game and also defines important terms of Game Theory such as Nash Equilibrium and best response. The Chapter 2.1.2 explains the extensive form games with their representation form of the game tree and the essential differences to the static form game. A further distinction within the Game Theory of zero- and general-sum games is made in Chapter 2.1.3 and Chapter 2.1.4. Chapter 2.1.5 further explains the difference between cooperative and non-cooperative games, whereupon Chapter 2.1.6 explains the distinctions and effects of information in games. Finally, Chapter 2.2 defines differential games as part of dynamic games.

2.1. Introduction to Game Theory

Game Theory is a mathematical theory concerned with the study of strategic decision making among players, who are often called agents (Zanardi et al., 2021a). The decision situations modeled by Game Theory are intended to help understand the phenomena that can be observed when players interact.

Definition 2.1.1 *Let i be a game-theoretical designation for a player and a set of players i are defined as $N = (1, ..., n)$. A player here can be an individual or a group of individuals (Holler et al., 2019).*

The models of Game Theory are abstract representations of real-world situations that, because of their abstractness, can study a wide range of phenomena (Osborne and Rubinstein, 1994; Haurie and Krawczyk, 2000). Thus, Game Theory provides a concept for analyzing game situations and formalizes the extent to which players form their expectations about the behavior of fellow players or react to the actions of fellow players and make the individual action. This formation of expectations and the comprehension of the decisions of the fellow players are central starting points of Game Theory (Holler et al., 2019). Within a game, players execute multiple moves. Thus, the overall progression of a game results from the sequence of moves executed by all players.

Definition 2.1.2 *A decision or a move is defined as the executed action of a player i. A move is a single decision made by a player i at a particular time in the game (Winter, 2019).*

Borrowing the term from control theory, a move is also understood to be the realization of a player's control (Haurie and Krawczyk, 2000; Winter, 2019).

Definition 2.1.3 *By the term strategy s_i of a player i we understand a complete game plan for the entire game (Winter, 2019).*

Colloquially, the term strategy is often used in the context of long-term planning and smart decisions. In contrast, in Game Theory, strategy is referred to as the complete game plan for an entire game, completely independent of whether the game plan is good or bad. The task of Game Theory here is to distinguish the good strategies from the bad ones. A complete game plan, in terms of a player's game plan, means that for every decision problem that the player may face during a game, the player will find an action to be performed in the game plan. This complete game plan ensures that the player is prepared for all eventualities and that no situations can arise in which the player cannot apply the game plan (Winter, 2019; Holler et al., 2019). A game theoretic situation only becomes a decision problem for a player if the player has several strategies at the players disposal and thus has several possibilities to set up a game plan. To analyze a game theoretic situation it is therefore essential to determine the best strategy for the individual player. For this purpose, the set of all available strategies of a player is examined (Winter, 2019).

Definition 2.1.4 *The set of all available strategies s_i of a player i is called the individual strategy set S_i. From this strategy set $S_i = (s_1, ..., s_i, ..., s_n)$, the player i can choose between different strategies $s_i \in S_i$ (Holler et al., 2019).*

The rules of a game are indirectly captured and specified by the strategy set S_i (Holler et al., 2019). Provided that a player chooses exactly one strategy from the players individual strategy set, the player chooses a pure strategy.

Definition 2.1.5 *A pure strategy gives a complete definition of how a player will play a game. In particular, it determines the move a player will make in every possible situation (Holler et al., 2019).*

A pure strategy is often chosen with access to information available to a player at the time of the move. However, if the player uses a random mechanism to decide between pure strategies, then the player randomizes and chooses the so-called mixed strategy (Holler et al., 2019).

Definition 2.1.6 *A mixed strategy is a probability distribution that assigns to each available action a likelihood of being selected. If only one action has a positive probability of being selected, the player is said to use a pure strategy (Holler et al., 2019).*

Another way to choose a strategy is the behavioral strategy. This defines a random execution of an admissible move from the strategy set depending on available information. According to Kuhn and Tucker (1953), this strategy is closely related to mixed strategies and consequently for many games the concepts of mixed strategy and behavioral strategy coincide (Haurie and Krawczyk, 2000). To the extent that a player succeeds in choosing a strategy that is the best response to all the strategies of the other players that are possible at all, that player is playing a dominant strategy.

Definition 2.1.7 *A strategy is dominant if the payoffs a player can achieve with this strategy are basically higher than the payouts the player can achieve with any other of the players strategies, regardless of what actions the other players perform (Winter, 2019).*

Since the dominant strategy is a player's best possible strategy, it is understandable that this is precisely the strategy that should be chosen. Assuming that a player's strategy s_1 leads to a lower payoff than the player's strategy s_2 in every case, strategy s_1 is dominated by strategy s_2. Since the player with strategy s_2 has a strategy that dominates the other strategy s_1, there is no rational understandable reason for the player with strategy s_1 to continue to consider strategy s_1 (Winter, 2019). For this reason, a basic assumption of Game Theory is that players pursue defined rational goals and strategically adjust their knowledge and expectations to the behavior of other players.

Definition 2.1.8 *Rational choice theory states that individual players make rational calculations to make rational decisions and achieve outcomes that are consistent with their own personal goals. These outcomes are also associated with maximizing a player's self-interest. It is expected that the application of rational choice theory will lead to outcomes that provide the greatest utility and satisfaction to players given the limited options available to them. The personal goals of the players are not subject to qualitative (Osborne et al., 2004).*

Because each player i selects a strategy s_i that represents a complete game plan, the combination of one strategy per player results in a complete game plan for the respective game situation.

Definition 2.1.9 *If the strategy sets S_i from Definition 2.1.4 of all players i are combined, the strategy space S of the game is obtained. The strategy space $S = S_1 \times ... \times S_i \times ... \times S_n$ of an entire game is the Cartesian product of the strategy sets S_i of the individual players i (Winter, 2019).*

Provided that the players of a game situation can decide between pure and mixed strategies, the strategy space is represented according to the Definition 2.1.10.

Definition 2.1.10 *Let the strategy space for game situations with pure or mixed strategies be $\Sigma = \Sigma_1 \times \Sigma_2 \times ... \times \Sigma_n$ and the strategy set available to each individual player i defined as $\sigma = (\sigma_1, \sigma_2, ...\sigma_n)$ with $\sigma_i \in \Sigma_i$ and represented (Holler et al., 2019).*

The decisions in game situations, which a player makes on the basis of his strategy, are made under a certain uncertainty, since the player in most cases does not know which decisions the other players will make. Especially in simultaneous games, it becomes clear that it is not possible for the players to make their own decision depending on the decisions of the other players. Therefore, an essential aspect of Game Theory is that for these game situations the expectations of the players are analyzed and, if this is possible, strategic considerations can also be derived from them (Holler et al., 2019). The course of each game is therefore determined by a strategy set and thus also has an influence on the outcome of the game. The outcome of the game has consequences for each player, which can be evaluated both positively and negatively with numbers. The higher the number, the better the outcome of the game is evaluated by the respective player, because the players wants to maximize their rewards (Winter, 2019).

Definition 2.1.11 *Let payoffs J be the numbers, which evaluate the consequences of a game from the point of view of each player. In the context of Game Theory, the payoffs of an individual player are real numbers that contain the desirability of the possible outcomes of the game (Zanardi et al., 2021a; Haurie and Krawczyk, 2000).*

A general example from economics for payoffs is the amount of money a player can win or lose. Each player's payoff depends, among other things, on the actions of other players, so a rational decision by an individual player cannot be made without considering the other players (Zanardi et al., 2021a; Haurie and Krawczyk, 2000). Each player pursues the goal of maximizing his profit by playing a game whose objective function is called utility or reward, or minimizing it whose objective function is defined as cost function or loss (Zanardi et al., 2021a).

Definition 2.1.12 *If a certain strategy s_i of a player i is played in a game situation, the payoff function $J_i(s)$ results for this player i, which defines the payoff J_i depending on the strategy s_i Holler et al. (2019).*

In the context of this work, the payoffs J_i for each player i are formed considering the current states of the player i and their strategy s_i. The outcome of solving a game is to determine each player's strategy. The solver of a game tries to determine the expected sets of strategies and ideally instructs each player which strategy to choose. This is only possible if the players can rank the various events according to their preferences.

Based on the Definitions in this Chapter a game can be defined in general Game Theory according to Theorem 2.1.1.

Theorem 2.1.1 *A game is defined as $\Gamma = (N, S, J)$ is described by*

- *the set of players i $N = \{1, ..., n\}$,*
- *the strategy set $S_i = (s_1, ..., s_i, ..., s_n)$ for each player i with $s_{ij} \in S_i$ the j-th strategy of player i,*
- *the strategy space $S = S_1 \times ... \times S_i \times ... \times S_n$, which gives the set of all possible strategy combinations $s = (s_1, ..., s_i, ..., s_n)$ from each player's strategies such that $s_i \in S_i$ holds and*
- *the payoff functions $J = (J_1, ..., J_n)$. The payoff here is defined for player i as $J_i(s)$, provided that the strategy s is played.*

To illustrate the basic concept of Game Theory with an example, the decision problem is explained in more detail below using the Figure 2.1. In this maneuver, two players, vehicle 1 and vehicle 2, travel side by side on a two-lane highway. The set of the players can therefore be defined as $N = \{1, 2\}$. On the right lane, a blue obstacle is approaching in the direction of travel of the two vehicles. Player 1 has two decision selection strategies in this example and can choose the strategy $s_{11} = (remain)$ to stay on the lane and the strategy $s_{21} = (switch)$ to swerve to the edge of the lane. The player 2 also has two strategies to choose from. First, the player can choose the strategy to stay on the lane $s_{12} = (remain)$, and second, the player can switch to the left lane $s_{22} = (switch)$. Of course, the given strategies are only a selection to illustrate this example.

These strategies thus give the strategy sets $S_1 = \{s_{11}, s_{21}\}$ for player 1 and $S_2 = \{s_{12}, s_{22}\}$ for player 2. The two players must make a decision simultaneously without knowing which decision the other player has chosen. This procedure is called simultaneous or static games and is explained in more detail in Chapter 2.1.1. The result of each match is described by the cost, which includes the damage of each player. Figure 2.1 describes for each of the four situations the payoff function $J_i(s)$ for the player 1 with $J_1(s)$ and for the player 2 with $J_2(s)$. As an example, the first situation, which is visualized on the left in Figure 2.1, is explained below. Assuming that both players choose

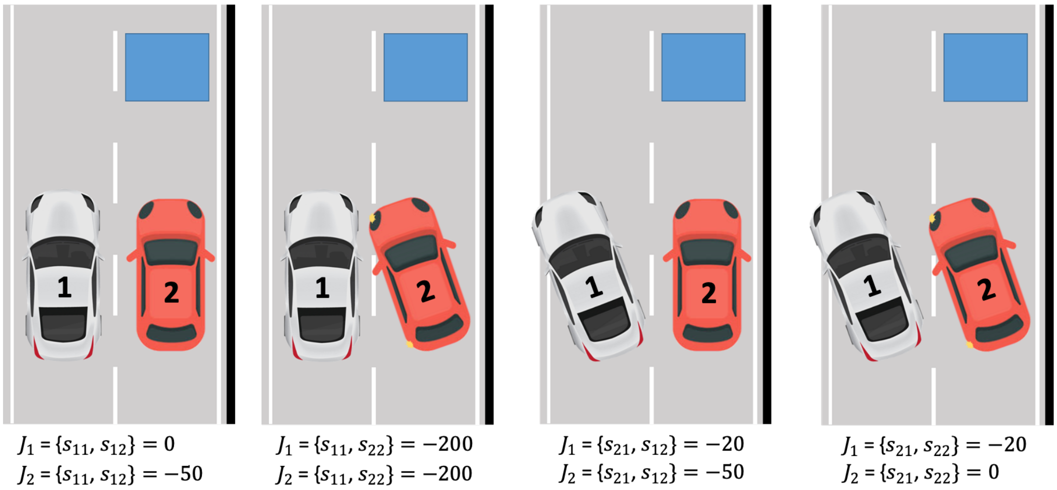

Figure 2.1.: Schematic representation of a decision problem to illustrate the basic concept of Game Theory

the strategy of staying on the lane, i.e. s_{11} and s_{12} are played, the payoff $J_1 = \{s_{11}, s_{12}\}$ results in 0 cost for player 1 when this strategy combination is played. The cost arises because the player 1 follows the lane and does not leave the lane, and also the player 2 does not drive into the lane of player 1. In contrast, the payoff of player 2 is $J_2 = \{s_{11}, s_{12}\}$, when this strategy combination is played, incurs a cost of -50. Since both players 1 and 2 stay in their lane, only player 2 incurs a negative cost because he collides with the blue obstacle.

2.1.1. Static form games

The representation form of a game by the tuple $\Gamma = (N, S, J)$ is also called strategic form or static form (Hart, 1992).

Definition 2.1.13 *A static form game is a model of interacting decision makers for a static game in which all decision makers, usually referred to as players, and generally unaware of the actions of the other players, simultaneously choose their action (Ozdaglar, 2015; Osborne and Rubinstein, 1994).*

This model assumes not only that all players act simultaneously, but also that players have only one turn, so that each player performs only one action (Winter, 2019). In general, static form games are represented in a matrix form and are defined by the fact that players cannot observe the actions of other players before performing their own actions because players act simultaneously (Holler et al., 2019). Since players choose their actions only once and players perform their actions simultaneously, so that no player can choose the action depending on the actions of the other players, time does not matter in static form games (Osborne and Rubinstein, 1994). Moreover, in static form games, the availability of information plays a crucial role in the outcome of the game and the payoff that each player receives. Therefore, in simultaneous games, complete and incomplete information is available and both the rules of the game and the payoff of the players are known to the fellow players (Holler et al., 2019). The possibilities of information in games are explained in Chapter 2.1.6.

In static form games with complete information, all players know exactly their own payoffs and the payoffs of all other players, as well as all possible outcomes associated with them. Thus, the other players are able to take the perspective of any other player and analyze the game from that player's point of view. In a static form game with complete information, a player can assume that not only the player himself, but also all the other players collectively know the player's best strategy and that the player will choose it to achieve the best possible payoff. In static form games with incomplete information, players know their own payoffs, but not all players know the payoffs of the other players and all possible outcomes. Accordingly, it is not possible for the players to take the other players' perspective exactly, so they cannot be absolutely sure what actions the other players will take. Therefore, the behavior of the other players is random to some extent. Therefore, the analysis of static form games with incomplete information tries to determine how players should deal with these random strategies of their fellow players (Winter, 2019).

In Figure 2.1 a static form game has already been introduced in which the players 1 and 2 execute their actions simultaneously. Within a static form game, the strategy space is often represented in the form of a matrix. Considering the strategy sets $S_1 = \{s_{11}, s_{21}\}$ for player 1 and $S_2 = \{s_{12}, s_{22}\}$ for player 2, the Cartesian product of the two strategy sets is the following strategy space S for the two players 1 and 2:

$$
\begin{array}{cc}
 & \begin{array}{cc} s_{12} & \qquad s_{22} \end{array} \\
S = & \begin{bmatrix} (s_{11}, s_{12}) & (s_{11}, s_{22}) \\ (s_{21}, s_{12}) & (s_{21}, s_{22}) \end{bmatrix} \begin{array}{c} s_{11} \\ s_{21} \end{array}
\end{array}
$$

The payoff combinations of the example on Figure 2.1, which result from the strategy sets of the two players 1 and 2, can also be expressed in the form of a matrix for a static form game:

$$\begin{array}{cc} & \begin{array}{cc} s_{12} & \qquad s_{22} \end{array} \\ S = & \begin{bmatrix} (0,-50) & (-200,-200) \\ (-20,-50) & (-20,0) \end{bmatrix} \begin{array}{c} s_{11} \\ s_{21} \end{array} \end{array}$$

From this matrix, all possible costs of the possible payoff combinations for the two players 1 and 2 can be derived. Furthermore, this matrix allows to determine a Nash Equilibrium (NE).

Definition 2.1.14 *A strategy combination is a Nash Equilibrium (NE) if for each player i and each strategy s_i of the player i, s is at least as good as the strategy combination (s_{i1}, s_{i2}) according to the preferences of player i, where player i chooses strategy s_i while the other player j chooses strategy a_j. Equivalently, for every player i,*

$$J_i(s) \geq J_i(s_i, s_{-i})$$

where J_i is a payoff function that represents player i's preferences (Osborne and Rubinstein, 1994).

A NE represents a situation in which each player maximizes the expected payoff given the actions of the other players (Myerson, 1984) and in which each has no incentive to deviate from the current strategy. Moreover, the NE describes the best payoff that each player could expect if they all behaved rationally (Zanardi et al., 2021a). Provided that player W expects player 2 to behave rationally and therefore chooses the NE strategy, it is best for player 1 to also choose the NE strategy (Jørgensen and Zaccour, 2003). In Game Theory, the concept of NE is an essential concept for analyzing the outcomes of strategic interactions between two or more players (Ramachandran and Tsokos, 2012). Due to the different interests of the players, not all players may prefer the same strategy combination and accordingly prefer a different combination (Winter, 2019). According to Definition 2.1.14, an optimal strategy combination represents a situation in which no player in a game has an incentive to deviate from the chosen strategy.

Definition 2.1.15 *The best answer is the strategy of a player that ensures the player at least the same or a higher payoff against the given strategies of his other players (Winter, 2019). We denote by $R_i(s_{-i})$ the set of actions that yield the best possible outcome to player i, when the other players play strategies s_{-i} (Zanardi et al., 2021a):*

$$R_i(s_{-i}) = arg\,max\, J_i(s_i, s_{-i})$$

Considering the example from Figure 2.1, the best responses according to Definition 2.1.15 can be defined for player 1 and player 2. The best responses for player 1 to the strategies of player 2 are the following:

- If 2 chooses strategy s_{12}, the best response of 1 is to choose strategy s_{11}.
- If 2 chooses strategy s_{22}, the best response of 1 is to choose strategy s_{21}.

The best responses for player 2 to player 1's strategies are as follows:

- If 1 chooses strategy s_{11}, the best response of 2 is to choose strategy s_{12}.
- If 1 chooses strategy s_{21}, the best response of 2 is to choose strategy s_{22}.

From the matrix of the example in Figure 2.1 and based on the defined best responses for each player, two NE can be identified. Both the strategy combination $\{s_{11}, s_{12}\}$ with outcome (0, -50) and the strategy combination $\{s_{21}, s_{22}\}$ with outcome (-20, 0) are NE for both players 1 and 2 in this example, since neither player would choose a different strategy after the fact, as their payoffs would worsen or remain the same.

2.1.2. Extensive form games

In Game Theory, besides static games, there are also dynamic games with sequential structure. A strategy then consists of the definition of a certain sequence of moves, in which individual moves are often planned in dependence on preceding actions of the other players. This allows the players who move later to take into account the current state of the game when choosing their actions. For this reason, the strategies of these players are defined at the beginning of the game taking into account if-then instructions (Winter, 2019; Holler et al., 2019). In Chapter 2.1.1 it has already been explained that the static form of a game is generally best represented in the form of a matrix. While representing dynamic sequential games in the form of a matrix is possible and useful for some analysis, representing a game in a matrix does not take into account the timing of a game and, furthermore, can only take into account two players at a time in the form of a player or team (Zanardi et al., 2021a).

Definition 2.1.16 *Let a game in extensive form be a specification of a game in Game Theory that allows an representation of the sequence of possible moves of all players, their decisions at each decision point, the information each player has about the moves of the other player when the player makes a decision, and the players payoffs for all possible game outcomes (Osborne and Rubinstein, 1994).*

In the extensive form of a sequential game, in contrast to the representation in a matrix, the strategic interaction can be specified more precisely, in that the extensive form represents what actions the individual player performs at a particular time in the game and what information the player has at that time. A game in extensive form, which is a convenient representation of multilevel games (Zanardi et al., 2021a), can be conveniently represented by a game tree. The game tree is a generalization of a decision tree for multiple players (Kockesen and Ok, 2007).

Theorem 2.1.2 *Let a game tree $G = (V, E)$ be a graph in which V represent the vertices and E represent the edges. V is partitioned among players such that $V_i \subset V$ are the vertices in which player i decides and $V = V_1 \cup V_2 \cup \ldots \cup V_n$ holds. An edge represents a vertex which has no vertices as successors. Edges are labeled to indicate which action has been chosen by which player at this point.*

A game tree, as visualized by the Figure 2.2, evolves from the root of the tree on the left to the leaves of the tree on the right. Each vertex of the tree corresponds to the point at which the player must decide which action to choose. Accordingly, the edges between the vertices represent the possible moves of the individual player whose turn it is. Vertices represent situations within a game in which players must decide on an action or choose a strategy. Accordingly, vertices are labeled with the name of the player whose turn it is at that moment. The root as the beginning of the game tree is called the initial vertex and is to be distinguished from the other vertices. Formally, this difference is expressed by representing the initial vertex as an open circle. All other decision vertices are represented as closed circles. In contrast, end vertices represent the end of the game, so that no player performs another action. Behind each end vertex, a payoff vector is created that contains the payoffs for the players of the sequential game, so that it can be read directly which entries of the payoff vector the respective players will receive when reaching the end vertex. The order of payoffs here correspond to the order in which the players chose their action. Starting from each decision vertices, the edges of the tree connect the decision vertices or end vertices with each other. Each edge represents an action that the player can choose at a decision vertex. Each edge is further labeled with the action it represents (Kockesen and Ok, 2007). As explained earlier, the game proceeds in time from initial vertices on the left to final vertices on the right. Thus, decision vertices that lie on top of each other belong to the same point in time and to the same player. For this reason, especially in more complex sequential games, it makes sense to number and label the decision and end vertices and to include them in the representation of the game tree (see Figure 2.2) (Winter, 2019).

To explain extensive form games, we again use the example from Figure 2.1. In the context of extensive form games, players act sequentially rather than simultaneously. In this example, both players 1 and 2 have perfect and complete information (cf. Chapter 2.1.6). Player 2 starts and executes the first action. The player can choose the strategy $s_{21} = (remain)$ to stay in the lane and

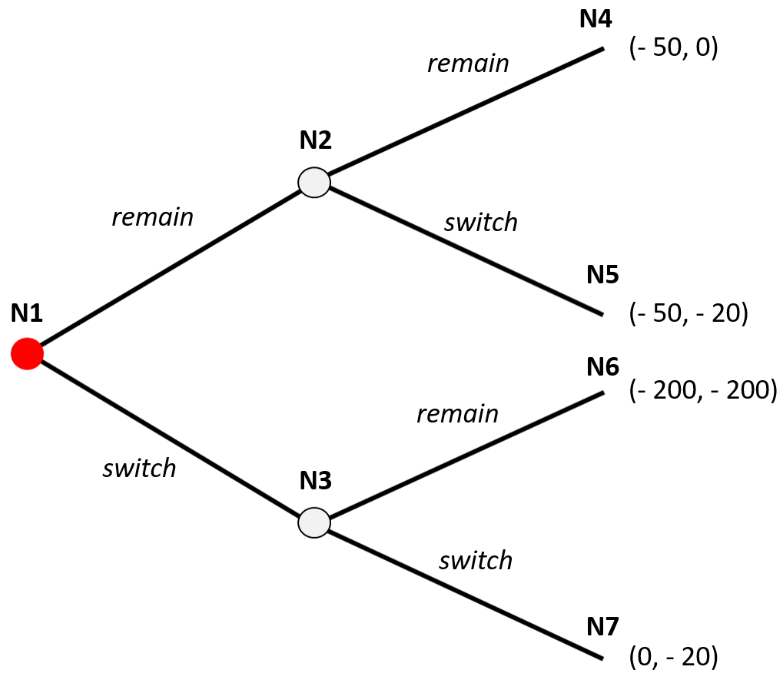

Figure 2.2.: Game tree of an extensive form game for the example from Figure 2.1

the strategy $s_{22} = (switch)$ to switch to the left lane. This results in the following strategy space for player 2:

$$S_2 = (s_{12}, s_{22}) = (remain, switch) \tag{2.1}$$

At the beginning of the game, player 1 has the possibility to choose from four different strategies. These strategies take into account that player 1 has no information about the other player's action. One option for player 1 is to always stay in lane $s_{11} = (remain, remain)$ regardless of which strategy player 2 chooses. Also independent of the strategy of player 2 is the second strategy $s_{12} = (switch, switch)$ of player 1, where the player always chooses to leave the lane to the left. In the third strategy $s_{13} = (remain, switch)$, player 1 leaves the lane only when player 2 switches to the left lane, so this strategy depends on the previous action of player 2. Also, the strategy $s_{14} = (switch, remain)$ depends on the previous action of player 2, since it implies that player 1 switches lanes only if player 2 stays in the lane. These four strategies form the following strategy space S_1 for player 1:

$$S_1 = \{s_{11}, s_{21}, s_{31}, s_{41}\} \tag{2.2}$$

The formulated game as an extensive form game is visualized in the form of a decision tree in the Figure 2.2, in which player 2 must make a decision at the red vertex $N1$ and player 1 must make decisions at the white vertices $N2$ and $N3$. Taking the payoff functions and their real values from the example in the context of Figure 2.1, the payoff functions of the extensive-form game can be represented in a matrix. This matrix is spanned in the rows by the strategies of player 2 and in the columns by the strategies of player 1.

$$S = \begin{array}{cc} \begin{array}{cccc} s_{11} & s_{21} & s_{31} & s_{41} \end{array} & \\ \begin{bmatrix} (-50,0) & (-50,-20) & (-50,0) & (-50,-20) \\ (-200,-200) & (0,-20) & (0,-20) & (-200,-200) \end{bmatrix} & \begin{array}{c} s_{12} \\ s_{22} \end{array} \end{array}$$

As with the static form games in Chapter 2.1.1, the best answers for the extensive form games can be defined according to Definition 2.1.15 for player 1 and player 2. Since player 2 acts first, the best response for the player in this example is to choose the strategy s_{22}, since this allows the player to avoid a collision with the obstacle. The best responses that player 1 can give to the strategies of player 2 in this game are the following:

- If player 2 chooses strategy $s_{12} = (remain)$, the best response of player 1 is to choose strategy s_{11} or strategy s_{31}.
- If player 2 chooses strategy $s_{22} = (switch)$, the best response of player 1 is to choose strategy s_{21} or strategy s_{31}.

The definition of the best answers for the two players can be understood using the matrix for this game. Based on the defined best responses for each player, two NE can be identified. Both the strategy combination $SC_1 = \{s_{21}, s_{22}\}$ with the outcome of (0, -20) and the strategy combination $SC_2 = \{s_{31}, s_{22}\}$ with the same outcome of (0, -20) are NE for both players 1 and 2 in this example, since neither player would choose a different strategy in hindsight as their payoffs would worsen or remain the same. However, the strategy combination SC_1 is not credibly tractable because, from a rational point of view, player 1 has no interest in choosing strategy $s_{12} = (switch, switch)$ in this example. It makes no sense from the point of view of player 1 to switch to the roadside, and negative costs are incurred if player 2 does not switch. Therefore, only the strategy combination SC_2 can be considered a plausible NE and outcome of the extensive form game.

Game trees, by their basic illustration of the dynamic structure of the game, provide a simple way to express games in detail and to analyze them (Gibbons et al., 1992). Moreover, the order of the sequence of moves is particularly emphasized by the representation with a game tree. However, a disadvantage of representing games by game trees is that the size of the game tree can quickly become large even for simple games. By the example of a chess game it becomes clear that a complete description of a chess game by the extensive form will be hardly presentable in its complexity due to the multiplicity of possibilities (Haurie and Krawczyk, 2000). In addition, the matrix representation is better at identifying equilibria than the game tree representation. However, the matrix representation is limited to two players, whereas the game tree representation is not (Winter, 2019).

2.1.3. Zero-sum game

Theorem 2.1.3 *Zero-sum games describe situations and game events in which the sum of the payoff of all players is equal to zero, whereby the payoffs balance out accordingly (Ferguson, 2014). A zero-sum game is a 2-player game in which (Zanardi et al., 2021a):*

$$J_1(s_1, s_2) = -J_2(s_1, s_2),\ \forall s \in \Gamma$$

According to Theorem 2.1.3, any gain in payoff by one player simultaneously leads to a loss in payoff by the other player (Ferguson, 2014). Zero-sum games, such as races, like Figure 2.3, are therefore inherently counterproductive and are generally indicated by a single outcome of the game. Accor-ding to Zanardi et al. (2021a), several practical constellations exist in which the structure of zero-sum games is justified. On the one side, the already explained game situation in which the players compete against each other and which therefore end with a winner and a loser. On the other side, game situations in which nature represents the adversary in order to achieve robustness of the player decision against worst case scenarios. An example of this is the Liniger and Van Gool (2020) formulation of the curvature of a road that a vehicle is driving towards, which is considered an adversarial action of the vehicle. An example of conversion to equivalent zero-sum games are safety games, as formulated by Korzhyk et al. (2011).

If a zero-sum game is a finite game, it can be formulated as a matrix game (Ferguson, 2014) and if it is a two-person game, a single matrix can be used due to the zero-sum formation, so that zero-sum games are generally also called single-matrix-games (Kuhn, 2009). In the following, the Theorem 2.1.3 will be explained by means of a race example. For this purpose, as in the example from Figure 2.1 before, the two players 1 and 2 are modeled as vehicles. The two players have the possibility to choose two possible strategies for the race, which are shown in the matrix below.

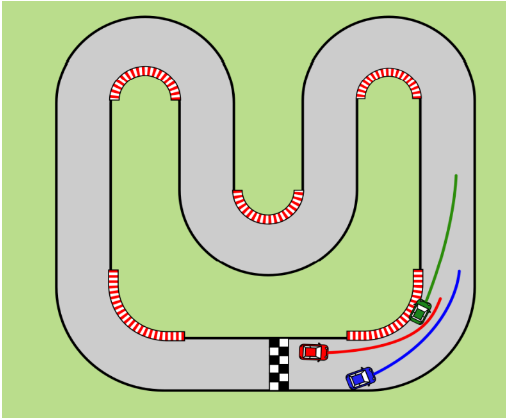

Figure 2.3.: Representation of a zero-sum game using the example of a race Zanardi et al. (2021b)

Each row represents one of the two strategies $s_1 = (s_{11}, s_{21})$ that player 1 can choose and each column represents one of the two strategies $s_2 = (s_{12}, s_{22})$ that player 2 can choose. As can be seen from the matrix, the sum of the payoffs of both players under the condition of having chosen one of the two strategies is always zero, since $J_1(s) = -J_2(s)$ holds.

$$S = \begin{array}{cc} \begin{array}{cc} s_{12} & s_{22} \end{array} & \\ \begin{bmatrix} (1,-1) & (-1,1) \\ (-1,1) & (1,-1) \end{bmatrix} & \begin{array}{c} s_{11} \\ s_{21} \end{array} \end{array}$$

Since players in zero-sum games pursue opposite preferences, such as winning a race, there is no interest for players to act as a coalition. It can be concluded that zero-sum games are non-cooperative games (see Chapter 2.1.5) and therefore each player must choose his strategies and actions in uncertainty about the opponent's behavior (Bacharach, 1989). In addition to modeling for two players, a zero-sum game can also be modeled for two teams (see Chapter 2.1.5). In this game, the players are divided into group A and group B and the payoff depends on the actions of the players from the teams and is positive for each player from group A in the case of a win, so correspondingly the payoff of each player from group B is negative due to the loss. Provided there is perfect coordination within a group, this zero-sum interaction between two groups is in principle nothing more than a zero-sum interaction between two players (Schulman and Vazirani, 2017). This principle can be applied to the Figure 2.3, since in this example only one team can emerge as the winner of a race.

2.1.4. General-sum game

Zero-sum games from Chapter 2.1.3 contrast with general-sum games, such as city traffic, which generally do not form zero-sums due to the complexity caused by the variety of individual vehicles and strategies. Zero-sum games, such as the racing scenario in Figure 2.3, furthermore do not take into account the description of the interaction between multiple players within a system. Using an everyday traffic situation as an example, it is clear that players have coupled goals, such as avoiding collisions or maintaining a safe distance, in addition to their personal goals, such as the destination or travel time (Zanardi et al., 2021a). Since these goals usually do not sum to zero, these games are called general-sum games or bimatrix games.

Definition 2.1.17 *The bimatrix game is a two-player game in normal form, where*

- *player* 1 *has a finite strategy set* $S_1 = \{s_{11}s_{21}...s_{n1}\}$,
- *player* 2 *has a finite strategy set* $S_2 = \{s_{12}s_{22}...s_{m2}\}$,
- *when the strategy combination* (s_{i1}, s_{j2}) *is chosen, the payoff for player* 2 *is* $J_2(s_{i1}, s_{j2}) \in \mathbb{R}$ *and the payoff for player* 1 *is* $J_1(s_{i1}, s_{j2}) \in \mathbb{R}$ *(Gokulraj and Chandrashekaran, 2021).*

The values of payoff functions can be described by a bimatrix:

$$\begin{array}{cc} \begin{array}{cccc} s_{11} & s_{21} & ... & s_{n1} \end{array} & \\ \begin{bmatrix} (a_{11}, b_{11}) & (a_{12}, b_{12}) & ... & (a_{1n}, b_{1n}) \\ (a_{21}, b_{21}) & (a_{22}, b_{22}) & ... & (a_{2n}, b_{2n}) \\ ... & ... & ... & ... \\ (a_{m1}, b_{m1}) & (a_{m2}, b_{m2}) & ... & (a_{mn}, b_{mn}) \end{bmatrix} & \begin{array}{c} s_{12} \\ s_{22} \\ ... \\ s_{m2} \end{array} \end{array}$$

The values of the payoff functions can be specified separately for specific players, where matrix W is called the payoff matrix for player 1 and matrix R is called the payoff matrix for player 2.

$$\mathbf{W} = \begin{bmatrix} a_{11} & a_{12} & ... & a_{1n} \\ a_{21} & a_{22} & ... & a_{2n} \\ ... & ... & ... & ... \\ a_{m1} & a_{m2} & ... & a_{mn} \end{bmatrix}$$

$$\mathbf{R} = \begin{bmatrix} b_{11} & b_{12} & ... & b_{1n} \\ b_{21} & b_{22} & ... & b_{2n} \\ ... & ... & ... & ... \\ b_{m1} & b_{m2} & ... & b_{mn} \end{bmatrix}$$

A special case of this type of game is the so-called win-win game. Here all players can win at the same time. As the Figure 2.4 shows, this outcome is not automatically the case, so that other outcomes of the game are also possible in which only one player wins the game.

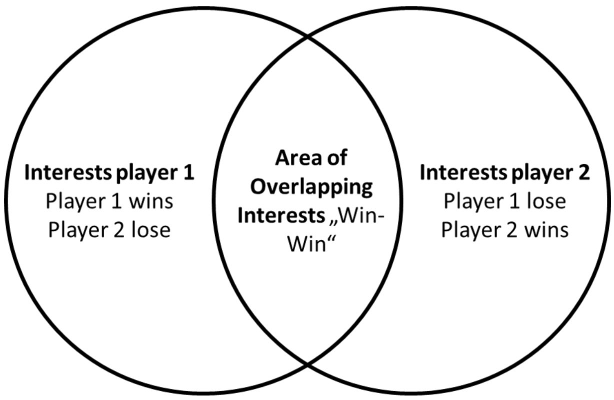

Figure 2.4.: Schematic illustrating zero-sum, general-sum and win-win games

2.1.5. Cooperative and non-cooperative games

Game Theory distinguishes, among other things, between the two basic types of cooperative and non-cooperative games.

Definition 2.1.18 *A cooperative game is a game with competition between groups of players, named coalitions, due to the possibility of external enforcement of cooperative behavior (Khan and Ahmad, 2006).*

Cooperative games, because of their game structure, allow players to communicate and plan as a group before choosing their next action. In contrast, this possibility is not present in non-cooperative games (Nash, 1950; Khan and Ahmad, 2006). Thus, in Game Theory, an important distinction is based on the ability of players to enter into a cooperative agreement or not. In a cooperation agreement, decision making is done collectively so that each player receives a profit to the maximum extent possible. This is done without creating inefficiencies within a game system.

The theory that deals with, among other things, coalition building or bargaining issues is defined as cooperative Game Theory (Zanardi et al., 2021a). As already explained, non-cooperative Game Theory is the counterpart to cooperative Game Theory.

Definition 2.1.19 *In non-cooperative games, each player acts independently without cooperation or communication with the other participants and makes decisions individually based on the information available to the player (Nash, 1951).*

The player carries out the decision process until he has no incentive to change the decision he has made (Nash, 1951; Zanardi et al., 2021a). Provided there is no incentive to change the decision made by the players in a game, equilibrium is reached. According to Nash (1950), for games with a finite number of available actions, there is always at least one NE (Zanardi et al., 2021a). However, a non-cooperative game cannot always be solved, so only partial solutions exist. These possess many properties of solutions, but in contrast are not unique. However, if a solution for non-cooperative games exists, it is unique (Nash, 1951).

Cooperative games, which are also called coalition games, have a collection of common action sets that each coalition, which is a group of players, can perform independently of the other remaining players. The forming specification of the coalition and the common action that a coalition performs are the outcomes of a coalition game. Furthermore, the theory of cooperative games is based on the preferences of individual players, even when the actions are performed as a coalition. A solution concept for coalition games assigns to each game a set of outcomes with which the players of a coalition game come to terms, so that they exhibit some stability. By satisfying the stability requirement, outcomes are generally immune to deviation by a particular type of group of players. Solutions of non-cooperative games, in contrast, require immunity to deviation by individual players (Osborne and Rubinstein, 1994).

The difference between a cooperative model and a non-cooperative model is that the focus is not on the potential of the individual players, but on the potential that the players in a group can achieve. If coalition formation is modeled within a non-cooperative game, it is necessary to specify to what extent coalitions form and to what extent coalition members can choose common actions, since this information is missing in a cooperative game, so that the outcome of a cooperative game does not depend on (Osborne and Rubinstein, 1994).

Based on the schematic representation of a decision situation visualized in Figure 2.1 in the Chapter 2.1.6, the differences between the two modeling approaches of Definition 2.1.18 and Definition 2.1.19 of cooperative and non-cooperative games shall be explained. Both players want to reach their individually defined destination without collision. Due to the obstacle in the lane of player 2, the player is forced to change lanes to the left. Each of the players has information to perform their own individual driving task to a specific destination, which includes driving in the left lane past the obstacle. However, the information of a single player is not enough to reach the defined destination without collision. Only with the information of the other player a collision-free handling of the driving task is possible. To ensure that both vehicles reach their destination without collision, the vehicles must cooperate with each other. In the case of a non-cooperative game model, the situation determines exactly what actions each player can perform. For example, in this situation, each player can provide its information for a consideration, such as a certain price, so that this information can be purchased and each individual player can reach its destination collision-free by purchasing the information. A coalition model, on the other hand, assumes the set of payoff vectors that a group of players can collectively achieve. Here, a coalition can establish binding agreements to ensure a collision-free driving task. As the explained situation in this Chapter shows, cooperative and non-cooperative play formulate the different types of strategic thinking and contribute to the understanding of strategic thinking (Osborne and Rubinstein, 1994).

2.1.6. Possibilities of information in games

The information structure of a game indicates to what extent the individual players of a game have certain information at their disposal. It distinguishes between the information that the individual player has at the beginning of the game and the information that the player has during the players moves. Information at the beginning of a game can be divided into complete and incomplete information.

If a player has complete information, the player knows which other players are playing this game, which actions are available to these players and to the player himself and which results are possible with these actions for each individual player (Haurie and Krawczyk, 2000). Here, the player is fully informed about all relevant characteristics of the fellow players, so that no other player has private information about individual characteristics. Thus, under the conditions of a game with complete information, a player can determine the optimal actions of the other players without observing the players' moves, making the analysis of such a game relatively easy. Moreover, the example of a card game shows that some aspects of real games are not considered in games with complete information, since uncertainty about the cards of the fellow players is a part of many card games. A game by a player not having the necessary information, such as preferences of the other players, is defined as a game with incomplete information (Holler et al., 2019).

In addition to the information available to a player at the beginning of the game, a distinction can also be made in information available to a player when he decides to make a particular move.

If the player has the knowledge of all preceding moves of the other players, the player has perfect information. In some game situations, however, this knowledge is not available, so the player has only imperfect information (Holler et al., 2019). The problem of a game with imperfect information within an extensive form game is explained in more detail with an example in Chapter 2.1.2.

However, as Harsanyi (1967) has shown, a game of incomplete information can be treated formally like a game with complete but imperfect information without difficulty. In a game of imperfect information, although certain actions of the players are not observable, everyone knows the structure of the game and all the properties of the players. It is also known what alternative actions the other players have and how they evaluate the alternatives.

In the course of a sequential game (cf. Chapter 2.1.2), the players of the game get new information about the moves of the other players at each turn. As far as a player remembers already acquired information, the players information becomes more accurate. If a player can remember all the information at each of the players decision points, concerning both the players teammate and the players own moves, the player has perfect recall and can make an accurate statement at the end of the game about what event occurred (Holler et al., 2019). If this is not the case, it is a game of imperfect recall for this player (Haurie and Krawczyk, 2000).

2.2. Differential games

In Game Theory, differential games are part of the general class of dynamic games. Dynamic games are mathematical models that represent the interaction between different players controlling a dynamic system over time (Haurie and Krawczyk, 2000).

A simple general example is the board game chess. The game of chess is a strategic board game between two players who take turns moving their pieces, also called chessmen, on the board, the chessboard. The winner is the player who manages to checkmate his opponent. Checkmating means attacking the king's piece in such a way that the opposing player can neither defend nor escape. In dynamic systems, players can influence by their actions the temporal evolution of the state of a system, which in the example of chess could be the positions of the other pieces on the chessboard. In dynamic games, as in chess, the difficulty lies in the action decision of the players. Each action of a player is influenced by the previous action of the other player and also influences the reaction of the following player (Haurie and Krawczyk, 2000). Differential games are a mathematical theory that deals with conflict problems modeled as game problems in which the behavior of players and their interaction with each other is described by differential equations

(Zanardi et al., 2021a).

Definition 2.2.1 *According to Bressan (2010) let $x \in \mathbb{R}^N$ describe the state of the system, evolving in time t according to the ordinary differential equation*

$$\frac{dx}{dt} = \dot{x} = f(t, x, u_1, u_2) \; t \in [0, T],$$

with initial data

$$x(0) = x_0.$$

Here $u_1(\cdot)$, $u_2(\cdot)$ are the controls implemented by the two players. We assume that they satisfy the pointwise constraints

$$u_1 \in U_1, u_2 \in U_2,$$

for some given sets $U_1, U_2 \subseteq \mathbb{R}^m$. For $i = 1, 2$, the goal of the i-th player is to maximize his own payoff, namely

$$J_i(u_1, u_2) \doteq \psi_i(x(T)) - \int_0^T L_i(t, x(t), u_1(t), u_2(t)) \; dt.$$

Here ψ_i is a terminal payoff, while L_i accounts for a running cost. Differential games, which can also be called state-space games, involve multiple state variables that can describe the state of a dynamical system $\frac{dx}{dt}$ at a given time t (Jørgensen and Zaccour, 2003). The study and mathematical formulation of differential games can be traced back to the work of Rufus Isaacs in 1954 in the mathematics department of the research and development cooperation (Ramachandran and Tsokos, 2012), which was followed by further work by Leitmann and Liu (1974), Krasovskii and Subbotin (1988) and Basar and Olsder (1995). Well-known examples of differential games include certain types of fights, such as an airplane being chased by a missile, or the conflict between workers and capitalists (Isaacs, 1999). From the examples given, it can be seen that differential games is a suitable discipline of applied science, which can analytically model conflict problems of the real world (Lewin, 2012). This type of games is based on the assumption that the past actions of the players and all general influences of past events are summarized in the current state variables Jørgensen and Zaccour (2003).

Theorem 2.2.1 *Let a state variable be one of the variables used to describe the mathematical state of a dynamical system or the state of a variable at a particular time T.*

By selecting a control variable of a player, the values of certain other variables, such as the speed or acceleration of a vehicle, are influenced within a differential game. These values of the other variables, which are changed by the control variables, are called state variables. Thus, the control variables are functions of the state variables. State variables change during the course of the game and, through their values at any point in time, reflect the current state of the game, which is thereby completely determined (Isaacs, 1999; Ramachandran and Tsokos, 2012). In reality, there may be inequality constraints on the state or control variables, which may also be referred to as admissible controls and trajectories. In addition, constraints on the final state may also be imposed (Starr and Ho, 1969b; Friedman, 2013). t_f represents the final time point on the differential games in the context of differential games, which may be variable or defined as a fixed time point (Starr and Ho, 1969b). Essentially, a differential game is a game in extensive form, see Chapter 2.1.2, which is played in continuous time, and in its analysis it has turned out to be advantageous to consider this game in the normal form representation (Jørgensen and Zaccour, 2003).

Since differential Game Theory is strongly related to optimal control theory, differential Game Theory can be described as a mixture of the notions of control theory and decision structures derived from classical Game Theory and the associated solution concepts (Lewin, 2012). The book by Krasovskii and Subbotin (1988) gives a detailed overview of the connection between control theory and differential Game Theory. According to Ramachandran and Tsokos (2012), it is natural to view a differential game as a control process in which the players among whom controls are divided are willing to commit to conflicting goals. The difference between optimal control theory and differential games is essentially that the optimal control problem considers only a single control. Differential games, therefore, cannot be reduced to optimal control models, since here the assumption is made that only one player actively participates in the action, while the other does not. In differential Game Theory each player tries to control the state of a system in such a way that the player reaches his goal or maximum payoff (Kamien and Schwartz, 2012). According to Ramachandran and Tsokos (2012), differential games can also be referred to as a class of two-sided optimal control problems. Thus, optimal control theory can be called merely a special case of differential Game Theory (Kamien and Schwartz, 2012). This special case was already described in 1962 in the work of Pontryagin et al. (2018) in terms of minimization problems, which can be formulated as differential games with one player. Among others, Kelendžeridze (1961) extended the work of Pontryagin et al. (2018) to two players. However, especially in a differential game of two or more players shows that the calculation of the NE is time consuming and especially in nonlinear games complex calculations must be made (Mylvaganam et al., 2017; Başar, 1986).

3. State of the art

Differential games for the use of motion planning can already be traced back to LaValle (1995) for autonomous robotics. However, insofar as robots leave the confined domain of a factory floor and enter the complex real world, a systematic and rational type of interaction is necessary (Jafary et al., 2018). Accordingly, autonomous players must explicitly consider other players in their decision making, so that in the context of autonomous driving, for example, a vehicle first predicts the actions, based on reasoning and uncertainty, of other vehicles before planning its own action according to them. Liniger and Van Gool (2020), Spica et al. (2020), Wang et al. (2019) and Williams et al. (2018) have studied racing scenarios in their work, see Chapter 2.1.3, that use game-theoretic principles to achieve interactive behaviors.

Outside of racing scenarios, Zanardi et al. (2021c) describe in their work the interactive game theoretic behaviors in a so-called Urban Driving Game (UDG). UDGs are a class of differential games for modeling urban driving interactions and incentives. The work classifies the UDGs as general-sum games, see Chapter 2.1.3, which have been formalized with a concise and comprehensive structure suitable for describing everyday driving interactions from a game-theoretic perspective. In the driving game UDG, a player's personal cost depends only on the player's individual state and the actions of other players. Each player within a UDG thus has his own priorities, driving style, and abilities, which are considered personal characteristics. In addition to personal interests, players also have a communal interest in not colliding with each other, which provides a certain structure in urban traffic interactions. Depending on the activation limits and the dynamics of the players, the preference of each agent is expressed through a lexicographic relation that includes the common goal of not colliding with other agents, which allows the principle of minimal violation to be introduced. The principle of minimal violation planning admits only the least bad and no infeasible solutions to the competitive game-theoretic path planning problem (Tmová et al., 2013; Wongpiromsarn et al., 2021). Conside-ring a civilized driving task that should involve collision-free driving, the cost function of the UDG will have a lexicographic ordering that first consists of not colliding and second consists of minimizing the individual personal cost. The work divides the lexicographic goals into the following three preferences:

1) Avoid collisions with other vehicles and the environment

2) Compliance with traffic regulations

3) Consideration of personal comfort and desired travel speed

The first preference of collision costs is set by the requirement to stay on the road with a width of 7.0 m and by the violation of the minimum safety distance of 4.0 m. The compliance with traffic rules is set in the second preference in terms of speed limit and lane limit. The third preference is presented as a weighted sum of secondary personal goals. The weighted sum consists of maintaining lane center, maintaining desired speed, and preferring comfort. Maintaining lane center is penalized quadratically by deviating longitudinally and laterally from the desired lane. In the course of maintaining the desired speed, each vehicle travels at a speed of 8.3 m/s at the beginning of the UDG, and each vehicle tries to maintain this speed. Also in the course of maintaining the desired speed, a deviation is penalized quadratically. However, this penalty is asymmetric, as speeds lower than the intended desired speed are penalized less. Preference for comfort is provided by a quadratic penalty for changes in steering speed and for fuel consumption. Zanardi et al. (2021c) also prove in their paper that UDGs can have an ordinal potential structure in the lexicographic sense, given the assumptions of common collision costs and personal goals. According to the authors, the potential structure enables consideration of iterated-best-response algorithms and computations of socially efficient NE via a single optimization problem. A socially efficient NE is a NE of society, taking into account all external costs and benefits as well as internal costs and benefits. The representative results of the paper are empirically illustrated by the agents' game-theoretic decision making in a challenging intersection example. In this example, players naturally exhibit reasonable rational behaviors derived on lexicographic preferences.

Figure 3.1 shows an excerpt of the simulated results of the work of Zanardi et al. (2021c). The left scenario in Figure 3.1 shows that the red vehicle is forced off the lane by the stationary black vehicle. The red vehicle abandons the goal of maintaining cruising speed to satisfy the lexicographical higher goal of collision-free travel with other vehicles or the environment. In contrast, in the right-hand scenario, the red vehicle is pushed into an aggressive driving maneuver that involves significant steering effort and lateral deviations. This impacts and drives up the cost to comply with traffic laws. The calculation of the NEs of the simulations shows that all NEs are admissible in the lexicographic sense regardless of order and method, and thus no equilibrium dominates the other equilibrium. A NE is admissible if no Nash Equilibrium dominates another NE in the lexicographic sense These equilibria are referred to as socially efficient NEs. By proving the existence of socially efficient NE, the work of Zanardi et al. (2021c) shows that efficient partitioning of mobility space can be possible in theory as well as in practice.

Within differential Game Theory, a framework has been introduced for studying problems in

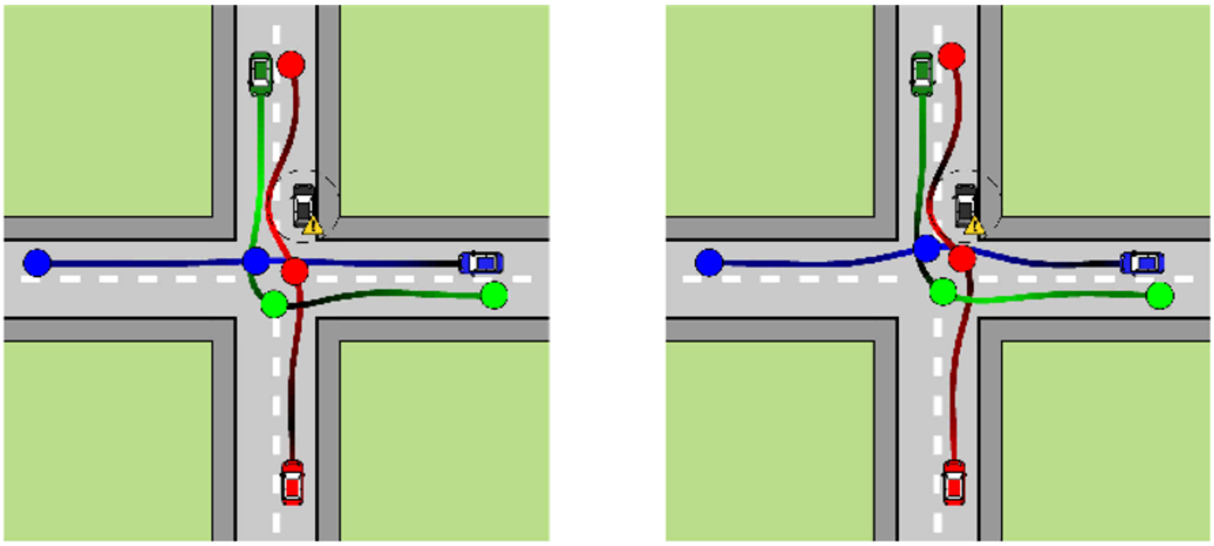

Figure 3.1.: Excerpt of the simulated results from the paper by Zanardi et al. (2021c)

which multiple players attempt to achieve their individual goals. These goals may compete with each other, but do not have to (Baqar and Olsder, 1982; Starr and Ho, 1969b,a; Isaacs, 1999). Arslan et al. (2007) therefore conclude that differential Game Theory can be useful for studying and solving problems in systems with multiple players. (Mylvaganam et al., 2017) consider a team of mobile agents in their work. They focus on the problem of steering these agents from their given initial positions to a set of predefined targets. The goal is to avoid collisions with static obstacles as well as collisions with other agents. This problem is already called the multi-agent collision avoidance problem in Mylvaganam et al. (2014) and is formulated in (Mylvaganam et al., 2017) as a differential game. Multi-agent systems originated in control engineering (Mesbahi and Egerstedt, 2010; Lewis et al., 2013; Leonard, 2013), where the accomplishment of a team consisting of agents of complex tasks is one of the main motivations within this research area of control engineering. Multi-agent systems address many application areas. In general, agents are expected to solve a task jointly. Some research topics dealing with multi-agent systems are inspired by naturally occurring systems such as fish schools, migratory birds, and bee swarms (Haque et al., 2011; Leonard and Fiorelli, 2001; Ogren et al., 2004, 2002; Su et al., 2009; Nabet et al., 2009; Paley et al., 2007). In Mylvaganam and Astolfi (2012, 2014), the problem of continuous monitoring of a defined region using teams consisting of unmanned aerial vehicles was formulated as a differential game, for which approximate solutions were found using the methodology developed in Sassano and Astolfi (2012). However, in addition to problems in which agents collaboratively solve a problem in a multi-agent system, there are also problems in which agents have individual and or conflicting goals. In the work of Mylvaganam et al. (2017), a multi-agent system consisting of N agents is considered. This system is a nonlinear differential game for which feedback NE solutions are sought. Since these equilibrium solutions can only be obtained by solving complex coupled partial differential equations and are not readily available, Mylvaganam et al. (2017) only consider approximate solutions, the computation of which has already been explored in the work of Mylvaganam et al. (2014). Using this solution, it can be shown that the strategies guarantee that the agents reach their goals while not causing collisions, provided that certain assumptions are met.

4. Methodological approach

Within these Chapters the methodological basis, which is necessary for the understanding of this work, is explained. Besides the ideation, which explains the basic idea of this work in Chapter 4.1, the modeling of the vehicle dynamics in Chapter 4.2 and the method for modeling a systematic description of scenarios in Chapter 4.3, which serve as the basis of the simulation, are explained within this Chapter. In addition, Chapter 4.4 provides an overview of how the reachable states of the players, which are used as vehicles in this work, can be determined using Continuous Reachability Analyzer. The Chapter 4.5 also explains how the cost functions are optimized in the context of this work and how worst cases can be considered within a defined game scenario.

4.1. Ideation

As already explained in Chapter 2.2, the solution of differential games is complex and time-consuming. Because of this, discrete control functions are used in this work instead of the continuous control function as they are generally used in differential games. For the different discrete control variables of the discrete control functions, the reachability analysis is used to define the different reachable states of the player. The reachability analysis makes it possible to take uncertainties into account. Based on the achievable states, the costs for each player can be estimated. It is interesting to note here that a state is not represented by a single vector, but by a geometric object. The reason for using geometric objects instead of vectors is that uncertainties in the state can be accounted for. Using the Continuous Reachability Analyzer (cf. Chapter 4.4), one can basically take an initial set represented by a geometric object, a system dynamics, and a time horizon, and compute all the states that are reachable from the initial state within the time horizon, given the system dynamics. Based on the discrete control functions it is possible to consider and solve normal form games instead of the complex and time consuming solution process of differential games.

4.2. Modeling vehicle dynamics

The vehicle dynamics are modeled according to the Kinematic Single-track Model (KS) by Althoff and Würsching (2020). Within the KS model, no rolling dynamics are considered, which allows to model a simplified vehicle using only two wheels. Here, both the front wheels and the rear wheels are combined as a wheel pair (Rajamani, 2011). Furthermore, the single-track kinematic model does not consider tire slip, so the velocity vector v, as visualized in Figure 4.1, is aligned at the center of the rear axle with a connection between the front and rear axles (Althoff and Würsching, 2020). The KS model is also used in the work of Paden et al. (2016) and Petti and Fraichard (2005), among others.

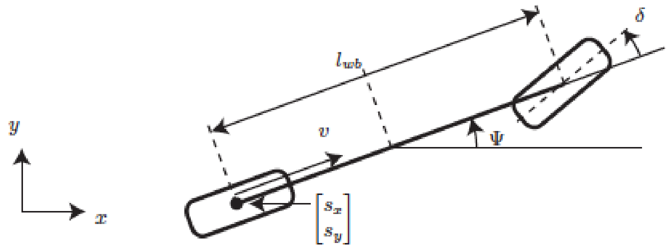

Figure 4.1.: Kinematic Single-track model according to Althoff and Würsching (2020)

$$x_1 = s_x \tag{4.1}$$

$$x_2 = s_y \tag{4.2}$$

$$x_3 = \dot{s}_x \tag{4.3}$$

$$x_4 = \dot{s}_y \tag{4.4}$$

Besides the variables of a point-mass model in Formula 4.5 to Formula 4.9 according to Althoff and Würsching (2020), the steering angle δ, the velocity of the steering angle v_δ and the wheelbase parameter l_{wb} are considered for the KS model. Accordingly, the differential equations of the KS model are defined as follows (Althoff and Würsching, 2020):

$$\dot{\delta} = v_\delta \tag{4.5}$$

$$\dot{\psi} = \frac{v}{l_{wb}} \cdot \tan(\delta) \tag{4.6}$$

$$\dot{v} = a_{long} \tag{4.7}$$

$$\dot{s}_x = v \cdot \cos(\psi) \tag{4.8}$$

$$\dot{s}_y = v \cdot \sin(\psi) \tag{4.9}$$

For the description of the KS model in terms of a state space, the following state variables in Formula 4.10 to Formula 4.14 and input variables in Formula 4.15 to Formula 4.16 are introduced (Althoff and Würsching, 2020):

$$x_1 = s_x \tag{4.10}$$

$$x_2 = s_y \tag{4.11}$$

$$x_3 = \delta \tag{4.12}$$

$$x_4 = v \tag{4.13}$$

$$x_5 = \psi \tag{4.14}$$

$$u_1 = v_\delta \tag{4.15}$$

$$u_2 = a_{long} \tag{4.16}$$

Provided the state and input variables of Formula 4.10 to Formula 4.16 are substituted into the constraints for steering, velocity, and acceleration in Formula 4.5 to Formula 4.9, the following equations result with which the dynamic of the KS model can be modeled (Althoff and Würsching, 2020):

$$\dot{x}_1 = x_4 \cos(x_5) \tag{4.17}$$

$$\dot{x}_2 = x_4 \sin(x_5) \tag{4.18}$$

$$\dot{x}_3 = f_{steer}(x_3, u_1) \tag{4.19}$$

$$\dot{x}_4 = f_{acc}(x_4, u_2) \tag{4.20}$$

$$\dot{x}_5 = \frac{x_4}{l_{wb}} \cdot \tan(x_3) \tag{4.21}$$

4.3. Method for modelling a systematic description of scenarios

The function development of automated driving functions of many projects and companies put a possible mass production of automated driving functions, which are tested by distance-based test procedures so far. According to estimates by Wachenfeld and Winner (2015), a required test distance of 6.22 billion kilometers is needed to verify an automated driving function with the Operational Design Domain (ODD) highway. Only after verification over this test distance could it be proven that the automated driving vehicle is twice as good as the human driver (DLR, 2019). The ODD can be defined according to the Society of Automotive Engineers (SAE): „Operating conditions under which a given driving automation system or feature thereof is specifically designed to function, including, but not limited to, environmental, geographical, and time-of-day restrictions, and/or the requisite presence or absence of certain traffic or roadway characteristics“ (SAE, 2018).

SAE (2018) introduced this concept to capture constraints for Level 1, 2, 3, and 4 driving automation. Level 5 driving automation describes the full driving automation and has an unlimited ODD that provides the same mobility as a human driving (SAE, 2018). Testing automated driving functions with a distance-based approach therefore involves a disproportionate amount of effort due to the large dimension of test kilometers. For this reason, new methods are needed for efficient testing and for verification and validation of automated driving functions. Thus, a new general state of the art for test methods and test case selection had to be defined, which can be used for a series release of these driving functions. The solution to this problem is the scenario-based approach for testing, verification and validation of automatic functions, which is also used for testing software. The scenario-based approach has the advantage of using a systematic and structured approach instead of a distance-based approach with random test cases. However, the change to a scenario-based approach raises new research questions regarding the level of performance expected from an automated driving system and the extent to which it can be verified that the desired performance can be consistently achieved (DLR, 2019).

The Research Project for the Establishment of Generally Accepted quality criteria, tools, methods and Scenarios and Situations (PEGASUS) for the release of highly automated driving functions addresses such research questions using the example of a highway driver ODD. The project is funded by the German Federal Ministry for Economic Affairs and Energy and includes, among other things, the sub project testing (DLR, 2019). Within the third sub-project Testing, a model for the systematic description of scenarios is defined. The model has been adapted for an automated representation of functional scenarios and a representation in an ontology based on previous work by Schuldt (2017). This model contains the following six independent Layers:

- Layer 1: Road-Level
- Layer 2: Traffic Infrastructure
- Layer 3: Temporary manipulation of Layer 1 and Layer 2
- Layer 4: Objects
- Layer 5: Environment
- Layer 6: Digital Information

As the Figure 4.2 visualizes, Layer 1 describes the road level, which includes the geometry and the topology of the road as well as the state and condition and the buildings and vegetation. Furthermore, the road marking is also taken into account. The second Layer defines the guiding infrastructure of the scenario through the signage in the form of danger and directional signs as well as the regulations that are regulated in the road traffic regulations (StVO, 2019; Böde et al., 2019).

Both Layer 1 and Layer 2 are defined according to the german guideline for the construction of freeways (FGSV, 2011). Temporary manipulations of Layer 1 and Layer 2 are conceptually described in Layer 3. This may take the form of temporary signage or marking lines to warn of or direct traffic through construction sites. Road user interactions through maneuvers are defined in Layer 4. In addition, the description of both static objects, such as parked vehicles, and dynamic objects is part of this layer. Examples of the dynamic objects are not only the road users, such as vehicles, but also the non-road users, such as animals or pedestrians. Layer 5 models the weather conditions of the scenario to be described. This includes parameters of the environmental conditions in the form of weather conditions of the road as well as lighting conditions and other

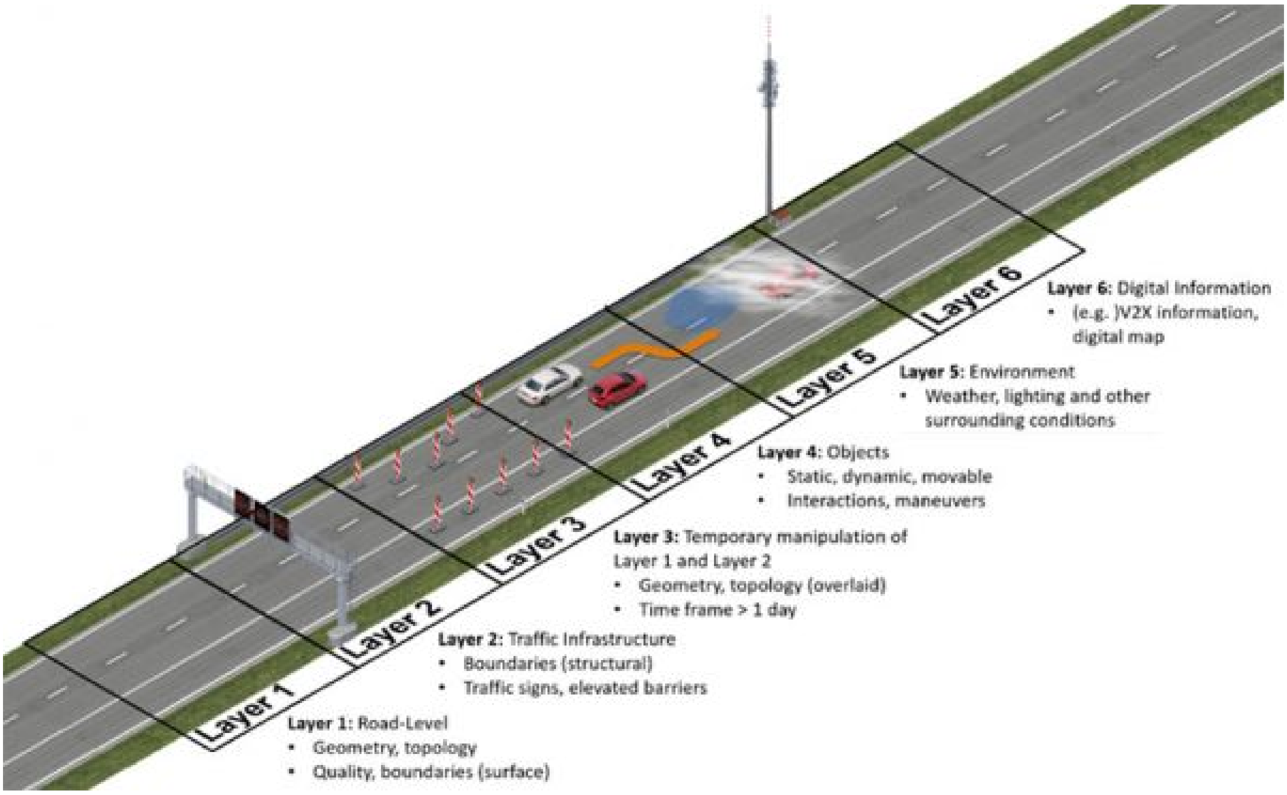

Figure 4.2.: Model for a systematic description of scenarios with six independent Layers DLR (2019)

particles or contaminants. Layer 6 describes the digital infrastructure and digital information, such as digital data or Vehicle-to-Everything. Among other things, the digital infrastructure provides road users, with a certain degree of automation, with information in the form of data via a cloud that provides the current location, other road users or possible hazards (Audi and Volkswagen, 2019).

4.4. COntinuous Reachability Analyzer (CORA)

In this work, the MATLAB toolbox CORA is used for the prototypical design of reachability analysis algorithms. CORA has been designed for various types of systems with purely continuous dynamics, such as linear systems, nonlinear systems, differential-algebraic systems, and parameter-variable systems, among others. The continuous part of the solution for a given discrete initial state is defined as

$$\chi(t; x_0, u(\cdot), p) \tag{4.22}$$

where $t \in \mathbb{R}$ is the time, $x_0 \in \mathbb{R}^n$ is the continuous initial state, $u(t) \in \mathbb{R}^m$ is the system input at $t, u(\cdot)$ is the input trajectory and $p \in \mathbb{R}^p$ is a parameter vector. Formula 4.23 defines the continuous reachable set at time t_f for a set of initial states x_0, a set of input values $U(t)$, and a set of parameter values P (Althoff et al., 2021).

$$R^e(t_f) = \{\chi(t_f; x_0, u(\cdot), p) \in \mathbb{R}^n | x_0 \in \chi_0, : u(t) \in U(t), p \in P\} \tag{4.23}$$

Since exactly reachable sets usually cannot be computed for system classes (Lafferriere et al., 2001) CORA supports the over-approximative computation of reachable sets. From this basis, CORA computes over-approximations for specific time points $R(t) \supseteq R^e(t)$ and time intervals $R(|t_0, t_f|) = \cup_{t \in |t0, tf|} R(t)$. CORA makes it possible to construct your own computation of the reachable set in a relatively short time (Althoff et al., 2021). These achievable quantities are the basis for the normal form game as well as their cost functions. In the context of this work, the basis for the cost functions is the mean value of the achievable set, which allows each achievable state to be represented as a point.

4.5. Optimization of the cost estimation

As already explained in Chapter 4.4, a set of reachable states is given by the CORA, which in turn are represented by geometric objects. In the context of this work, these are zonotopes. Since our defined cost functions do not deal with zonotopes, this work attempts to find representatives for each reachable set. Therefore, worst cases are estimated for each pair of controls included in the reachable states. Depending on the cost function, the cost functions are either maximized or minimized. In the context of the nonlinear optimization of the cost functions, optimization functions are defined in MATLAB, in which the respective cost functions are adapted with respect to the worst cases. Within an optimization function, a scalar optimization variable is created using the function $optimvar$ (see Formula 4.24), which creates expressions for the constraints $1, numconstr$ and the objective functions $LowerBound$ and $UpperBound$ of the optimization problem with respect to the variable x. The $UpperBound$ represents the largest possible and the $LowerBound$ the smallest possible value of a parameter.

$$x = optimvar(x, 1, numconstr, LowerBound, UpperBound) \tag{4.24}$$

Furthermore, an equation obj is required, which is to be optimized with respect to x in order to determine the worst case of the respective cost function for the calculation of the respective cost. If this equation is defined, it is defined to what extent this equation should be optimized (see Formula 4.25). $ObjectiveSense$ can be a structure with values minimize (*min*) or maximize (*max*).

$$prob = optimproblem(Objective, obj, ObjectiveSense, max/min) \tag{4.25}$$

The *solve* function can be used to solve problems ($prob$) defined in MATLAB for a variable x. In this case, the equation for a solution S can be defined according to Formula 4.26.

$$S = solve(prob, x) \tag{4.26}$$

Using this opimization function for the individual cost function, the initial states, which represent the worst case for the optimized function, can then be passed on to the cost function.

5. Creation of the normal form game

In the context of this work, a two player normal form game is defined, in which the two players are referred to as vehicles. This normal form game, whose structure is explained in Chapter 5.1, is a static, non cooperative, general sum game, in which the players do not receive any information about the preferences and properties of the other vehicle. With the help of this normal form game, the cost functions are to be evaluated on the basis of real scenarios with real maneuvers, which are defined in Chapter 5.2, and assessed with regard to their quality and correctness. With regard to the cost functions, a distinction is made between cost functions that are intended to avoid collisions in Chapter 5.3 and non-collision-avoiding cost functions from Chapter 5.4, which must nevertheless be taken into account due to their safety relevance.

5.1. Structure of the normal form game

The structure of the normal form game is significantly influenced by the initial state of the player i. In the context of this work, the players of a game are vehicles whose initial state can be defined based on the vehicle model from Chapter 4.2. The initial state for a vehicle i is defined as $state_{0,i}$ according to Formula 5.1.

$$state_{0,i} = [x_i, y_i, \delta_i, v_i, \psi] \tag{5.1}$$

For each parameter of the initial state of a vehicle $state_{0,i}$ the uncertainties can be quantified. Based on simulation results of the individual cost functions from the following Chapters 5.3 and 5.4, the following uncertainties for the initial state of a vehicle $state_{0,unc,i}$ can be quantified and assumed accordingly. Uncertainty for the parameter ψ does not need to be defined, since this parameter is not needed with respect to the cost functions and thus will have no impact on the cost of a game. For this reason, the parameter ψ will be considered further in the context of this work.

$$state_{0,unc,i} = [0.005, 0.005, 0.001, 0.001, 0.0] \tag{5.2}$$

In the context of this work, a strategy u is composed of the steering angular velocity ω and the acceleration a according to Formula 5.3.

$$u = [\omega, a] \tag{5.3}$$

Within a game, each vehicle i can choose between three strategies in the context of this work, so the strategies can be defined according to the following Formula 5.4.

$$u_i = [[\omega_{u1}, a_{u1}], [\omega_{u2}, a_{u2}], [\omega_{u3}, a_{u3}]] \tag{5.4}$$

Uncertainties can also be quantified for the strategies. Based on the simulation results of the individual cost functions from the following Chapters 5.3 and 5.4, the following uncertainties for the strategies of a vehicle $u_{unc,i}$ can be quantified and assumed according Formula 5.5.

$$u_{unc,i} = [0.001, 0.001] \tag{5.5}$$

Using the reachability analysis from Chapter 4.4, the states of a vehicle i over a defined time horizon $t_{horizon}$ are determined depending on the selected strategies u_i. For each state calculated by the reachability analysis over $t_{horizon}$, the costs J_i for the vehicle i are calculated as a function of the strategy u_i and added together for the defined time horizon $t_{horizon}$. Thus, costs J_i can also be incurred for a strategy, even if it does not yet incur costs at the beginning of the time horizon $t_{horizon}$. The cost J_i for the particular vehicle i is formed by the cost functions defined in the context of this work. These cost functions can be considered alone or directed weighted in interaction with other cost functions. The cost functions generate a payoff depending on the three strategies that each vehicle can choose. The payoff can be described and represented according to the Definition 2.1.17 by a 3x3 bimatrix for both vehicles.

$$\begin{array}{cc}
\begin{array}{ccc} \qquad u_{veh1,1} \qquad & \qquad u_{veh1,2} \qquad & \qquad u_{veh1,3} \qquad \end{array} & \\
\left[\begin{array}{ccc}
([\omega_{u1},a_{u1}],[\omega_{u1},a_{u1}]) & ([\omega_{u2},a_{u2}],[\omega_{u1},a_{u1}]) & ([\omega_{u3},a_{u3}],[\omega_{u1},a_{u1}]) \\
([\omega_{u1},a_{u1}],[\omega_{u2},a_{u2}]) & ([\omega_{u2},a_{u2}],[\omega_{u2},a_{u2}]) & ([\omega_{u3},a_{u3}],[\omega_{u2},a_{u2}]) \\
([\omega_{u1},a_{u1}],[\omega_{u3},a_{u3}]) & ([\omega_{u2},a_{u2}],[\omega_{u3},a_{u3}]) & ([\omega_{u3},a_{u3}],[\omega_{u3},a_{u3}])
\end{array}\right] & \begin{array}{c} u_{veh2,1} \\ u_{veh2,2} \\ u_{veh2,3} \end{array}
\end{array}$$

As explained in Definition 2.1.17, the payoff matrices for both vehicles can be specified separately, with matrix veh1 being the payoff matrix for vehicle 1 and matrix veh2 being the payoff matrix for vehicle 2.

$$veh1 = \begin{bmatrix}
([\omega_{u1},a_{u1}]) & ([\omega_{u2},a_{u2}]) & ([\omega_{u3},a_{u3}]) \\
([\omega_{u1},a_{u1}]) & ([\omega_{u2},a_{u2}]) & ([\omega_{u3},a_{u3}]) \\
([\omega_{u1},a_{u1}]) & ([\omega_{u2},a_{u2}]) & ([\omega_{u3},a_{u3}])
\end{bmatrix}$$

$$veh2 = \begin{bmatrix}
([\omega_{u1},a_{u1}]) & ([\omega_{u1},a_{u1}]) & ([\omega_{u1},a_{u1}]) \\
([\omega_{u2},a_{u2}]) & ([\omega_{u2},a_{u2}]) & ([\omega_{u2},a_{u2}]) \\
([\omega_{u3},a_{u3}]) & ([\omega_{u3},a_{u3}]) & ([\omega_{u3},a_{u3}])
\end{bmatrix}$$

Using the Python library Nashpy (Knight, 2017), which can be added to Python to compute equilibria in strategic 2-player form games, NE are determined based on the payoff matrices for both vehicles.

5.2. Modelling scenarios

According to Chapter 4.3, in this work, the cost functions are simulated, evaluated, and tested using a total of three scenarios, where Layer 3, Layer 5 and Layer 6 are not considered in the context of this work with regard to the modeling of the scenarios. These three scenarios should represent real scenarios between two vehicles, so that the simulation and evaluation of the cost functions is as close to reality as possible and can thus be transferred more easily into practice. Within these three scenarios, which will be defined in the following, reasonable realistic maneuvers of the two vehicles will be defined. By means of these maneuvers it shall be shown that the cost functions reflect the different costs depending on the strategies as it is to be expected for the respective maneuver within the scenario. For each scenario, there are of course more maneuvers in reality than defined in the scope of this work. Due to the complexity, this is only a selection of maneuvers that seem to make sense in the scenario under consideration. According to Chapter 4.3 the Layer 1, Layer 2 and Layer 4 can be defined identically for all three scenarios. Regarding Layer 1, the scenarios can be defined as two-way-traffic (twt) road, with a lane width of 3.5 m. Regarding the traffic infrastructure in Layer 2, a speed limit of 27.7778 m/s can be defined. Within Layer 4, from the point of view of a vehicle i, the other vehicle j can be defined as an object. In connection with the cost function obstacle distance defined in Chapter 5.3.5, an obstacle can be defined as an object, which will influence the trajectories of both vehicles on the road. In the following, among other Figures 5.1 to 5.3, the white vehicle is referred to as vehicle 1 and the red vehicle as vehicle 2.

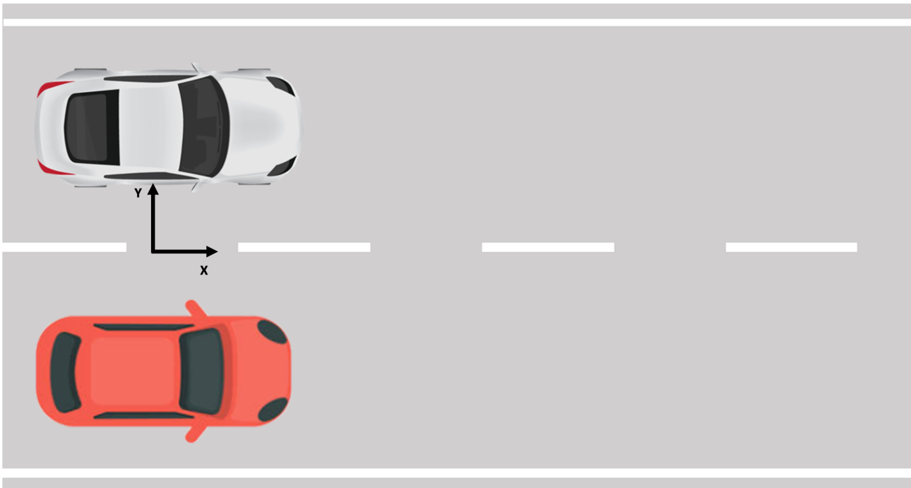

Figure 5.1.: Schematic representation of scenario 1

The first scenario, which is visualized in Figure 5.1, is supposed to represent an overtaking maneuver of the first vehicle, whose initial state is defined in Formula 5.6. The second vehicle, which travels at a lower speed than the first vehicle and whose initial state is defined in Formula 5.7, is thus overtaken. From the two Formulas 5.6 and 5.7, it can be seen that both vehicles are in the center lane of the respective lanes and there is a speed difference of 2.7778 m/s between the two vehicles.

$$state_{0,veh1} = [0.0, 1.75, 0.0, 27.7778, 0.0] \tag{5.6}$$

$$state_{0,veh2} = [0.0, -1.75, 0.0, 25.0, 0.0] \tag{5.7}$$

Since the first vehicle is on the other lane for overtaking, the only sensible maneuver for vehicle

1 is to complete the overtaking maneuver and change back to the previous lane. Vehicle 1 has to choose the strategy in such a way that it does not collide with the second vehicle and, if necessary, has to accelerate in order to change back to the previous lane. For vehicle 2, keeping the lane is a reasonable maneuver. In addition, the second vehicle could also increase speed and accelerate to the defined speed limit of 27.7778 m/s.

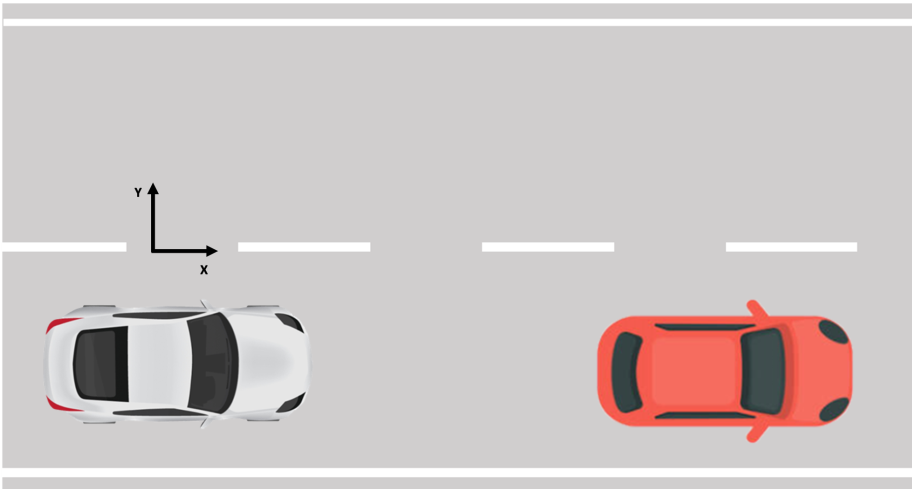

Figure 5.2.: Schematic representation of scenario 2

The second scenario, visualized in Figure 5.2, is intended to reflect the driving situation in which the first vehicle, whose initial state is defined in Formula 5.8, approaches at a higher speed a slower vehicle 2, whose initial state is defined in Formula 5.9. Identical to scenario 1, it can be seen from both Formulas 5.8 and 5.9 that both vehicles are on the lane center of the same lane and there is a speed difference of 2.7778 m/s between the two vehicles.

$$state_{0,veh1} = [0.0, -1.75, 0.0, 27.7778, 0.0] \tag{5.8}$$

$$state_{0,veh2} = [15.0, -1.75, 0.0, 25.0, 0.0] \tag{5.9}$$

As the first vehicle approaches the second vehicle with a speed difference of 2.7778 m/s, the first vehicle has the possibility to slow down and adapt to the speed of the second vehicle or to start an overtaking maneuver and overtake the second vehicle. A sensible maneuver for the second vehicle is, as in scenario 1, to stay in the lane or to increase the speed and to accelerate to the defined speed limit of 27.7778 m/s. The second vehicle can then overtake the first vehicle.

The third scenario, which is visualized in Figure 5.3, is intended to reflect a driving situation in which the first vehicle, whose initial state is defined in Formula 5.10, is approaching an oncoming

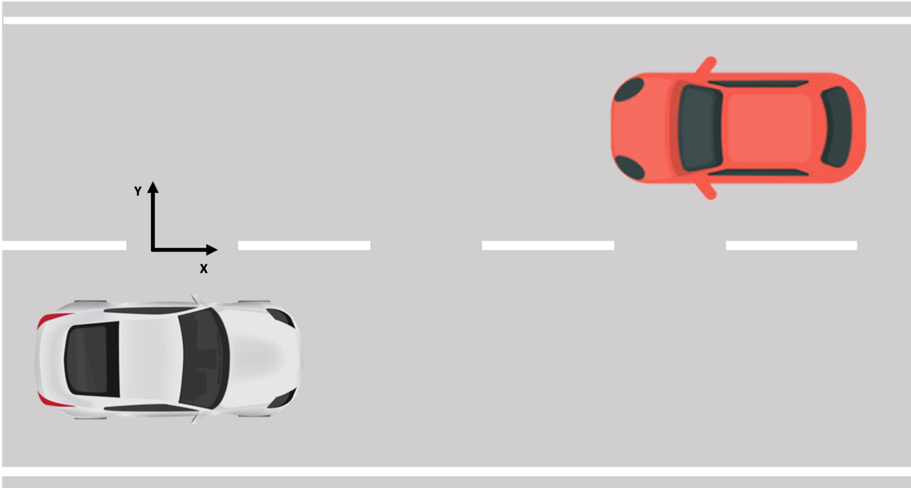

Figure 5.3.: Schematic representation of scenario 3

vehicle 2, which is driving in the other lane. The initial state of the oncoming vehicle 2 is defined in Formula 5.11. Identical to the other two scenarios, it can be seen from the two Formulas 5.10 and 5.11 that both vehicles are traveling on the lane center of the respective lane and there is a speed difference of 2.7778 m/s between the two vehicles.

$$state_{0,veh1} = [0.0, -1.75, 0.0, 27.7778, 0.0] \tag{5.10}$$

$$state_{0,veh2} = [60.0, 1.75, 0.0, 25.0, \pi] \tag{5.11}$$

Reasonable maneuvers for the third scenario are for both vehicle 1 and vehicle 2 to continue to follow their respective lane, since a change or departure from the lane may result in a frontal collision between the two vehicles. Vehicle 2 can also, as in scenarios 1 and 2, increase speed and accelerate to the defined speed limit of 27.7778 m/s as a result.

5.3. Modeling cost functions to avoid collisions

In the following Chapters the cost functions of collision energy in Chapter 5.3.1, Time-To-Collision in Chapter 5.3.2, center lane offset in Chapter 5.3.3, Euclidean distance in Chapter 5.3.4 and obstacle distance in Chapter 5.3.5 are explained. These cost functions are intended to avoid collisions and must therefore be considered for a safety analysis of a game.

5.3.1. Cost function collision energy

One way to estimate and calculate the severity of the accident is to calculate delta-V ($\triangle V$). This is a notation used in physics to denote the change in velocity of an object. In general, it is also used to refer to the change in velocity due to a collision. In terms of traffic accidents, $\triangle V$ refers to the change in velocity vector during a collision. When the magnitude and direction of the velocity changes rapidly, large forces act on the road user and these forces are expected to have a significant impact on personal injury. In addition, $\triangle V$ also takes into account the vulnerability of the road user. This property is important in studies of collisions because a light vehicle colliding with a heavy vehicle bounces back, whereas the speed of the heavy vehicle remains almost unchanged. An example of this is a collision between a truck and a car (Laureshyn et al., 2017). This assumption is supported by examples of Evans (1994), Gabauer and Gabler (2008) and Johnson and Gabler (2012) from crash safety research. In addition, the relationship between $\triangle V$ and the risk of serious injury is also confirmed by Johnson and Gabler (2012), Evans (1994) and Gabauer and Gabler (2008) as well as Ryb et al. (2007). Based on these factual circumstances, Joksch (1993) and Shelby et al. (2011) designate $\triangle V$ as the best single predictor of accident severity.

The notation $\triangle V$ has been used in a wide variety of studies. In Shelby et al. (2011) and Gettman et al. (2008), $\triangle V$ was integrated into the automatic conflict analysis algorithms of the Surrogate Safety Assessment Model to analyze the severity of traffic conflicts. Sobhani et al. (2011) and Sobhani et al. (2013) supplemented $\triangle V$ with the use of kinetic energy Laureshyn et al. (2017) extended the $\triangle V$ traffic conflict measure by integrating the proximity to an accident as well as the outcome severity for the case where an accident would have occurred. Based on the computation of $\triangle V$, Astarita et al. (2020), Astarita and Giofré (2019) and Alonso et al. (2020) propose this, based on vehicle trajectories, as a variety of safety indicators can determine the location of potential collisions but do not differentiate between the severity of collisions.

To estimate the crash severity, the velocities of two colliding vehicles before the collision v_{1s} and v_{2s} and the masses m_1 and m_2 of these vehicles at an arbitrary angle are used. s denotes, with respect to the velocity and in the following also to other parameter values, the respective values before the collision of the vehicles. In contrast, f denotes in the following the respective parameter value after the collision. According to Newton, the relative velocity in a one-dimensional collision of two bodies is proportional to the velocity of the bodies before the collision multiplied by the coefficient of elasticity of the two bodies:

$$v_{2f} - v_{1f} = -\varepsilon \cdot (v_{2s} - v_{1s}) \; with \; 0 \leq \varepsilon \leq 1 \tag{5.12}$$

In the Formula 5.12 it must be taken into account that the elasticity ε decisively influences the values of $\triangle V$, since this value tells how much kinetic energy the two colliding vehicles absorb during the collision. Here, $\varepsilon = 0$ denotes a plastic or also inelastic collision, in which the maximum possible energy is absorbed, causing the vehicles to stick to each other and move at the same speed after the collision (Szabo, 1972). According to Shelby et al. (2011), the reality value of the elasticity is in between, since in reality collisions have a value of the elasticity coefficient of 0.4 at lower speeds and 0.1 at higher speeds (Shelby et al., 2011; Nordhoff, 2005). For the implementation, $\triangle V$ is calculated as if this were a completely inelastic collision, hence $\varepsilon = 0$. For approximate calculations it is obvious to consider collisions as inelastic because of the lower computational effort. As a result, the velocity values are underestimated according to Shelby et al. (2011). This underestimation amounts from 1 percent to 30 percent (McHenry and McHenry, 1997).

The $\triangle V$ values to which the road users are subjected in a specific collision can be calculated by applying the law of conservation of momentum (Burg et al., 2017). In the present work, the momentum $\overline{\mathrm{P}}$ is expressed as a two-dimensional vector of the product of the mass and the velocity vectors of the two vehicles before the collision $\overline{\mathrm{v_{1s}}}$ and $\overline{\mathrm{v_{2s}}}$:

$$\overline{\mathrm{P}} = m_1 \cdot \overline{\mathrm{v_{1s}}} + m_2 \cdot \overline{\mathrm{v_{2s}}} \tag{5.13}$$

Formula 5.13 is valid for both elastic and plastic collisions. By making the assumption $\varepsilon = 0$, the following mathematical relation is obtained for a plastic collision:

$$m_1 \cdot \overline{\mathrm{v_{1s}}} + m_2 \cdot \overline{\mathrm{v_{2s}}} = m_1 \cdot \overline{\mathrm{v_{1f}}} + m_2 \cdot \overline{\mathrm{v_{2f}}} \tag{5.14}$$

The vehicles absorb the maximum possible energy in a plastic collision, which causes the vehicles to stick to each other and move at the same speed after the collision. Thus, the velocities of the two vehicles $\overline{\mathrm{v_{1f}}}$ and $\overline{\mathrm{v_{1f}}}$ are the same after the collision. Accordingly, this velocity will be referred to as $\overline{\mathrm{V}}$ in the following. Thus, Formula 5.14 can be rearranged as follows:

$$\overline{\mathrm{v_{1f}}} = \overline{\mathrm{v_{2f}}} \mathrel{\hat{=}} \frac{m_1 \cdot \overline{\mathrm{v_{1s}}} + m_1 \cdot \overline{\mathrm{v_{2s}}}}{m_1 + m_2} \mathrel{\hat{=}} \overline{\mathrm{V}} \tag{5.15}$$

The Formula 5.15 considers only straight central collisions. Considering the angle of incidence α, based on the Formula 5.15, the modified Formula 5.16 results:

$$\overline{\mathrm{V}} = \frac{1}{m_1 + m_2} \cdot \sqrt{(m_1 \cdot v_{1s})^2 + 2 \cdot m_1 \cdot m_2 \cdot v_{1s} \cdot v_{2s} \cdot \cos\alpha + (m_2 \cdot v_{2s})^2} \tag{5.16}$$

By calculating $\overline{\mathrm{V}}$ from Formula 5.15, the velocity changes for the two vehicles during the collision can be determined:

$$|\overline{\triangle \mathrm{v}_1}| = |\overline{\mathrm{V}} - \overline{\mathrm{v}_{1\mathrm{s}}}| \tag{5.17}$$

$$|\overline{\triangle \mathrm{v}_2}| = |\overline{\mathrm{V}} - \overline{\mathrm{v}_{2\mathrm{s}}}| \tag{5.18}$$

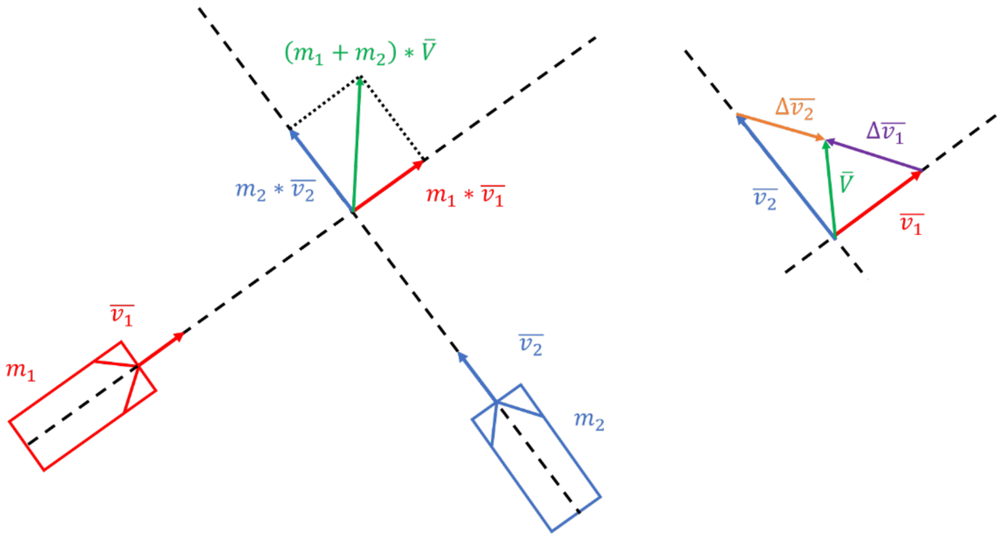

Figure 5.4.: Calculation of the resulting velocities $\overline{\mathrm{V}}$, $\overline{\triangle \mathrm{v}_1}$ and $\overline{\triangle \mathrm{v}_2}$ according to the law of conservation of momentum for elastic and plastic collisions by Astarita and Giofré (2019) and Sobhani et al. (2013)

As can be seen from the Formulas 5.17 and 5.18, in order to calculate the speed changes of the vehicles $\overline{\triangle \mathrm{v}_1}$ and $\overline{\triangle \mathrm{v}_2}$, it is necessary to calculate the differential speed of the vehicles as accurately as possible.

Also in expert opinions of accidents, the collision-related change in speed of the struck vehicle ($\triangle V$) is considered, among other things, as the decisive parameter. Including this parameter, it is often attempted to establish a relationship between the severity of the impact and the extent of a cervical spine injury (CSI) of the occupant (Dannert, 2005). A clear limit of a possible so-called harmlessness limit, which indicates the lower value up to which point a load is harmless and thus harmless, does not exist legally (Grönemeyer, 2008). Special importance is attached to the $\triangle V$-

range between 5 and 15 km/h in which partly no damage to the vehicles can be proved, because in this range non-structural CSI injuries cannot be diagnosed objectively (Elbel, 2007). In the studies of Eichberger et al. (1996), Eichberger et al. (1998), Kaneoka et al. (2002), Matsushita et al. (1994), Mertz and Patrick (1971) and Mühlbauer et al. (1999) the subjects described discomfort in the CSI area after sled tests, whereas in the studies of Eichberger et al. (1998), Kaneoka et al. (2002), Matsushita et al. (1994) and Mühlbauer et al. (1999) the $\triangle V$ was below 10 km/h during the tests. In the literature, different thresholds for $\triangle V$ are defined, from which a CSI violation is possible (Dannert, 2009; Elbel, 2007), assuming, that exceeding a harmlessness limit of 10 km/h for $\triangle V$ relevant damage can occur after passenger car rear-end collisions (Grosser, 2004; C, 1993; Lucka, 1998; Meyer et al., 1994; Miltner, 2002; Niederer et al., 2001). According to Weber et al. (2004) and Schmidt H and C (2004), a harmlessness limit between 20 to 30 km/h $\triangle V$ can be assumed for front collisions, and according to Becke et al. (1999), a harmlessness limit of 5 km/h $\triangle V$ can be assumed for side collisions.

To gain further insight into the problem of discomfort at low $\triangle V$, Meyer et al. (2002) a series of experiments in which subjects were simulated collisions using visual, auditory, and sensory tricks so that the $\triangle V$ was only 0 km/h. Despite the nonexistent $\triangle V$, 20 percent of subjects reported CSI injuries. Meyer et al. (2002) concludes that psychosomatic effects can play a significant role in collisions and that the effects can thus also occur in trials with $\triangle V$ psychosomatic effects. Furthermore, Sturzenegger et al. (1995) describes that different risks for CSI injuries can occur in relation to the seating position. For example, when the seating position is out of position (Fürbeth et al., 1999; Meyer et al., 1999), or when the direction of gaze is changed by moving the head (Winkelstein et al., 1999), the occupant's risk of injury can be significantly increased.

Based on the classification of the harmlessness limit in terms of $\triangle V$ by the literature, the following $\triangle V$ sectors for the collision energy cost function are defined in this work:

- Sector 1: 0 km/h < $\triangle V \leq$ 5 km/h
- Sector 2: 5 km/h < $\triangle V \leq$ 10 km/h
- Sector 3: 10 km/h < $\triangle V \leq$ 15 km/h
- Sector 4: 15 km/h < $\triangle V$

5.3.2. Cost function Time-to-Collision

According to Vogel (2003), the most effective method for measuring traffic conflicts is the use of safety indicators, since these indicators are promising tools for detecting risky rear-end collisions and for analyzing and evaluating traffic safety. In this context, the surrogate safety measures (SSM), which were developed based on the movement characteristics of vehicles, are suitable criteria for the definition of collision avoidance systems (Nadimi et al., 2016). In this context, SSMs represent indicators suitable for detecting hazardous situations (Archer, 2005; Barceló Bugeda et al., 2003; Cunto, 2008; Garber and Gousios, 2009; Gettman and Head, 2003; Sobhani, 2013; Young, 2014). So far, various safety indicators have been developed, such as Time-To-Collision (TTC), time after impact, unsafe density, collision avoidance deceleration rate, percentage of stopping distance, gap time, comprehen-sive time-based measure, rear-end collision probability, etc. (Hayward, 1971; Allen et al., 1978; Archer, 2005; Barceló Bugeda et al., 2003; Behbahani, 2014, 2015; Cooper, 1984; Cunto, 2008; Minderhoud and Bovy, 2001). Since the majority of SSMs use a driver and their response behavior as parameters to calculate the safety indicator, only TTC is considered in this work.

TTC is the time to collision and a main criterion in traffic conflict engineering, the concept of which was introduced by Hayward (1971) and Saffarzadeh et al. (2013) and which is applied to different types of conflicts such as rear-end, head-on and angle collisions (Minderhoud and Bovy, 2001). The TTC is the best known time-based safety indicator, which has been shown to be an effective indicator for evaluating traffic safety in traffic collision research (Minderhoud and Bovy, 2001; Vogel, 2003; Oh et al., 2006) and is also considered to be an effective measure for distinguishing between critical and normal behavior in rear-end collisions (Saffarzadeh et al., 2013). The TTC safety indicator is used, among other things, in the development of collision avoidance systems (Van Der Horst and Hogema, 1993), which can also be used in traffic management for highways (Balas and Balas, 2008). In the study by BMWI (2020), TTC was estimated in terms of length with the result that accidents in critical situations with a TTC of less than 1.3 s were hardly avoidable. In contrast, accidents in critical situations with a TTC of more than 1.7 s occurred very rarely. For this reason, the study sets the threshold of human driving ability in this range of 1.3 to 1.7 s. Accordingly, the higher the TTC value, the safer the situation. In the work of Lehsing (2019), situations with values smaller than 1.5 s were already classified as critical. In contrast, the TTC values of drivers who generally drove more cautiously were between 2.4 and 3.6 s.

Various improvements and concretizations have already been developed and proposed for the safety indicator. These include the introduction of the modified TTC (MTTC) (Ozbay, 2008), generalized formulations of the TTC (Saffarzadeh et al., 2013), developments of an inverse time to collision (Kiefer, 2005), and the TTC in terms of a moving section and a point (Laureshyn, 2010). However, these are not considered further in this work due to its complexity.

At a given time t, the TTC value thus defines the time required for two or more vehicles to collide under certain circumstances, provided that these vehicles continue to follow their current trajectory at their current speed and no evasive maneuvers are performed (Minderhoud and Bovy, 2001; Vogel, 2003; Lee, 1976). Using the Figure 5.5 and the following equation 5.19, the TTC value for a following vehicle F at a given time t can be calculated with respect to a preceding vehicle L (Saffarzadeh et al., 2013).

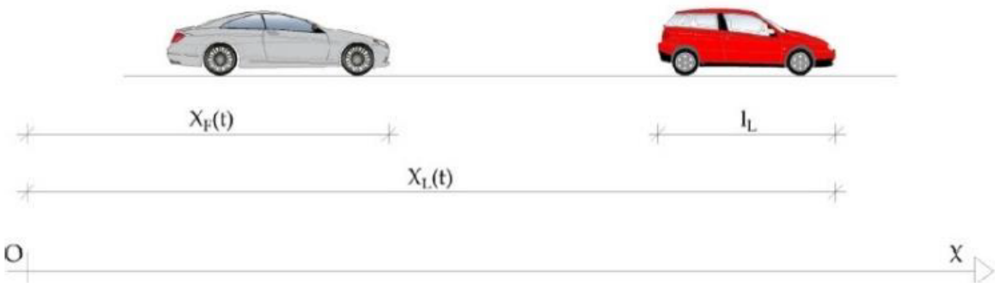

Figure 5.5.: Surrogate safety indicator TTC (Saffarzadeh et al., 2013)

$$TTC_F(t) = \frac{1X_L(t) - X_F(t) - l_L}{\dot{X}_F(t) - \dot{X}_L(t)} \qquad with \quad \forall \dot{X}_F(t) \, > \, \dot{X}_L(t \tag{5.19}$$

In this equation, X_L represents the position of the vehicle, the derivative $\dot{X}_L(t)$ of X_L after time t represents the velocity of the vehicle, and l_L represents the length of the vehicle L ahead. The following vehicle F is described by the position of the vehicle X_F, the velocity of the vehicle $\dot{X}_F(t)$ (Saffarzadeh et al., 2013). Since only the centers of gravity of the vehicles are considered in the context of this work, the length of the vehicle ahead l_L can be neglected, resulting in the following equation for the scope of this work:

$$TTC_F(t) = \frac{X_L(t) - X_F(t)}{\dot{X}_F(t) - \dot{X}_L(t)} \qquad with \quad \forall \dot{X}_F(t) \, > \, \dot{X}_L(t) \tag{5.20}$$

TTC thus represents, in the context of this work, the shortest distance between the two vehicles divided by the amount of relative speed of the two vehicles. Since the length of the vehicles is not considered, the TTC is the same for both vehicles. Since the TTC equation 5.19 is based on the assumption that both the vehicle ahead and the vehicle behind are traveling at constant speed until the collision, the actual acceleration or deceleration of the two vehicles during this period is not considered. Due to this, potential collisions may be incorrectly ignored due to discrepancies in acceleration and deceleration, resulting in some dangerous events that may affect traffic safety

(Saffarzadeh et al., 2013). Based on the Formula 5.21, the TTC in this work for the individual vehicle i depending on the other vehicle j is calculated as follows:

$$TTC_i(j) = \frac{X_j(t) - X_i(t)}{\dot{X}_i(t) - \dot{X}_j(t)} \tag{5.21}$$

Since at least 2 vehicles i and j are considered in the context of this work and according to Formula 5.21 the same costs for $TTC_i(j)$ and $TTC_j(i)$ would be determined for both vehicles, the quadrated speed v of the respective vehicle enters into the cost J. Thus, if the speed of a vehicle i is higher, the cost J_i for that vehicle is also higher.

$$J_i(j) = \frac{X_j(t) - X_i(t)}{\dot{X}_i(t) - \dot{X}_j} \cdot \dot{X}_i^2 = TTC_i(j) \cdot v_i^2 \tag{5.22}$$

$$J_j(i) = \frac{X_i(t) - X_j(t)}{\dot{X}_j(t) - \dot{X}_i} \cdot \dot{X}_j^2 = TTC_j(i) \cdot v_j^2 \tag{5.23}$$

According to Formula 5.22 and Formula 5.23, a value for the TTC for the individual vehicle is calculated at each point in time regardless of how far apart they are. Since the costs add up over the time period considered in the static form game, (see Chapter 5), this calculation is not useful for analyzing the resulting costs $J_i(j)$ and $J_j(i)$. For this reason, a safety distance for TTC $d_{TTC,safe}$ is introduced, so that only if the shortest distance $d_{TTC,s}$ between the two vehicles falls below the safety distance $d_{TTC,safe}$, the TTC for the respective vehicle is calculated.

5.3.3. Cost function center lane offset

The cost function center lane offset calculates the cost $J_{clo,i}$ for the amount of deviation from the center lane. To determine the amount of deviation from the center lane, the shortest distance $d_{clo,s}$ of the vehicle i to the center lane is calculated. Since, among other things, smaller deviations are hardly avoidable and since uncertainties with respect to the position of the vehicle are considered in the context of this work, a tolerable deviation from the center lane is introduced in the form of the distance $d_{tol,clo}$. According to Formula 5.24, this distance results from the lane width d_l and the center lane tolerable distance factor $K_{tol,clo}$. Figure 5.6 illustrates possible factors for the center lane tolerable distance factor $K_{tol,clo}$.

$$d_{tol,clo} = K_{tol,clo} \cdot d_l \tag{5.24}$$

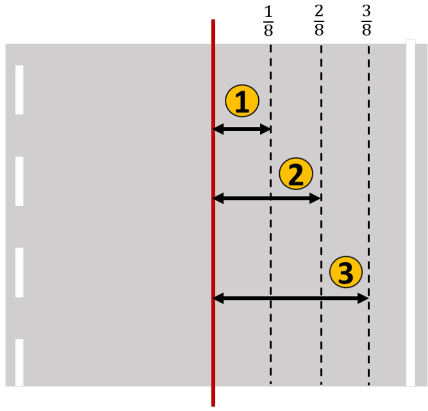

Figure 5.6.: Schematic representation of the tolerable distance from the lane center offset

For the cost function center lane offset, costs $J_{clo,i}$ are calculated for the respective vehicle i only if the shortest distance of the vehicle to the center lane $d_{clo,s}$ is smaller than the tolerable distance to the center lane $d_{tol,clo}$. The cost $J_{clo,i}$ for the particular vehicle i as a function of the tolerable distance $d_{tol,clo}$ according to Formula 5.24 and the speed v_i are calculated according to Formula 5.5.3.

$$J_{clo,i} = d_{cl,s} \cdot v_i^2 \tag{5.25}$$

The cost function center lane offset also distinguishes between one way traffic (owt) and two way traffic (twt). Since a deviation from the tolerable center lane distance $d_{tol,clo}$ can have more severe consequences for twt than for owt due to oncoming traffic, a factor is used to account for these cost differences. This factor $K_{twt,cl}$ is considered in the calculation for the cost $J_{clo,i}$ in Formula 5.5.3.

$$J_{clo,i} = d_{cl,s} \cdot v_i^2 \cdot K_{twt,cl} \tag{5.26}$$

5.3.4. Cost function Euclidean distance

The Euclidean distance is the distance term of Euclidean geometry. It is used to determine the distance between two points in a plane or in a space (Kowol, 2009). The Euclidean distance is used in the context of this work to determine the shortest distance $d_{eucl,s}$. In the euclidean distance cost function, the shortest distance $d_{eucl,s}$ between vehicle i and vehicle j is to be determined. Unless the vehicles are traveling on the same roadway, the cost $J_{eucl,i}$ of each vehicle i is calculated using

Formula 5.27. As formulated in Formula 5.27, the cost $J_{eucl,i}$ becomes higher as the respective vehicle i or vehicle j gets closer to the other vehicle. In addition, the squared velocity v of the respective vehicle enters the cost. Thus, if the speed v_i of a vehicle i is higher, the cost $J_{eucl,i}$ for that vehicle i is also higher compared to a slower vehicle at the same shortest distance $d_{eucl,s}$.

$$J_{eucl,i} = d_{eucl,s} \cdot v_i^2 \tag{5.27}$$

According to the Formula 5.27, a value for the distance to the other vehicle j is calculated for vehicle i at each time t, regardless of how far vehicle i is from vehicle j. Since the costs in the static form game (see the Chapter 5) sum over the given time horizon, this calculation is not useful for analyzing the resulting cost $J_{eucl,i}$. For this reason, a safety margin $d_{eucl,safe}$ is introduced so that only if this margin $d_{eucl,safe}$ is fallen short of, the cost $J_{eucl,i}$ is calculated with respect to the shortest distance d_s to the vehicle j for the respective vehicle i.

If the two vehicles are driving on the same lane, the direction in which the two vehicles are driving is taken into account. For this purpose, the steering angles δ_i and δ_j of the two vehicles are used. Since the steering angle is not expected to be exactly the same due to uncertainties and smaller steering movements, a steering angle tolerance is calculated for the two vehicles. This is composed of the lower bound in Formula 5.28 and the upper bound in Formula 5.29 of a vehicle i.

$$\delta_{LB,i} = \delta_i - range(\delta_i) \tag{5.28}$$

$$\delta_{UB,i} = \delta_i + range(\delta_i) \tag{5.29}$$

If the steering angle δ_i of vehicle i is within the steering angle tolerance of vehicle j and the steering angle δ_j of vehicle j is within the steering angle tolerance of vehicle i, it can be assumed that the two vehicles are moving in the same direction.

Assuming that the two vehicles travel on the same roadway and both the steering angle δ_i of vehicle i is within the steering angle tolerance range of vehicle j and the steering angle δ_j of vehicle j is within the steering angle tolerance range of vehicle i, the calculation of the cost J_{eucl} for the respective vehicle is distinguished in whether the shortest distance $d_{eucl,s}$ between the two vehicles is smaller than the previously defined desired parameter. Provided that the shortest distance $d_{eucl,s}$ is smaller than previously defined parameter, the cost is calculated by Formula 5.27.

5.3.5. Cost function obstacle distance

The cost function obstacle distance describes the costs $J_{obs,i}$ incurred for the vehicle i depending on the shortest distance $d_{obs,s}$ between the respective vehicle i and the obstacle. In the context of this work, an obstacle is understood to be Layer 4 from Chapter 4.3, in which static objects, such as parked vehicles or other objects, are defined, among others. As formulated in Formula 5.30, the cost $J_{obs,i}$ becomes higher the closer the respective vehicle i gets to the object. Moreover, we the goes the squared velocity v_i of the respective vehicle into the cost. Thus, if the speed v_i of a vehicle i is higher, the cost $J_{obs,i}$ for this vehicle is also higher compared to a slower vehicle at the same shortest distance $d_{obs,s}$.

$$J_{obs,i} = d_{obs,s} \cdot v_i^2 \tag{5.30}$$

According to the Formula 5.30, a value for the obstacle distance is calculated for the individual vehicle at each time point t, regardless of how far the vehicle is from the obstacle. Since the costs sum over the specified time horizon in the static form game (see the Chapter 5), this calculation is not useful for analyzing the resulting cost $J_{obs,i}$. For this reason, a safety distance $d_{obs,safe}$ is introduced so that only if this distance $d_{obs,safe}$ is undercut, the cost $J_{obs,i}$ is calculated with respect to the shortest distance $d_{obs,s}$ to the obstacle for the respective vehicle i.

5.4. Modeling other relevant cost functions

In the following Chapters the cost functions of quadratic velocity in Chapter 5.4.1, steering angle in Chapter 5.4.2 and acceleration in Chapter 5.4.3 are explained. These cost functions do not directly avoid collisions, but must be taken into account for a safety-related consideration of a game.

5.4.1. Cost function quadratic velocity

The basis for the cost function quadratic velocity is the defined speed limit v_L. Since in the context of this work also uncertainties and thus also uncertainties in the speed v are considered, it is meaningful for the speed limit v_L to consider both an upper bound v_{LUB} and a lower bound v_{LLB}. Considering the speed limit tolerance v_T, the following boundaries for the speed limit v_L result:

$$v_{LUB} = v_L + v_T \tag{5.31}$$

$$v_{LLB} = v_L - v_T \tag{5.32}$$

In the cost function quadratic velocity only the deviation from the speed limit v_L or the boundaries of the speed limits v_{LUB} and v_{LLB} leads to the cost of a vehicle i $J_{vel,i}$. For this reason, the boundaries of the speed changes $\Delta v_{LUB,i}$ and $\Delta v_{LLB,i}$ of the vehicle i versus the boundaries of the speed limit are calculated from Formula 5.31 and Formula 5.32.

$$\Delta v_{LUB,i} = |v_{LUB} - v_i| \tag{5.33}$$

$$\Delta v_{LLB,i} = |v_{LLB} - v_i| \tag{5.34}$$

The cost $J_{vel,i}$ for the respective vehicle i whose speed v_i is above the upper bound of the speed limit v_{LUB} is calculated according to Formula 5.35. If the speed v_i is below the lower bound of the speed limit v_{LLB}, the costs are calculated according to Formula 5.36 under consideration of a factor K_{vel}, which provides the under run of the lower bound of the speed limit with lower costs. If the speed v_i of the vehicle i is within the bounds, there is no cost for the vehicle i.

$$J_{vel,i} = \Delta v_{LUB,i}^2 \quad with \;\; v_i > v_{LUB,i} \tag{5.35}$$

$$J_{vel,i} = K_{vel} \cdot \Delta v_{LLB,i}^2 \quad with \;\; v_i < v_{LLB,i} \tag{5.36}$$

5.4.2. Cost function steering angle

In active steering systems, the system reduces the steering angle δ_i at high speeds, enabling better directional stability and safe steering movements that contribute to driving safety (Herold et al., 2008; Pfeffer and Harrer, 2013). This principle is also used to define the cost of the steering angle $J_{steer,i}$ in the context of this work. The higher the velocity v_i of the vehicle i, the higher the cost of the steering angle $J_{steer,i}$ as a function of the squared velocity v_i^2.

$$J_{steer,i} = |\delta_i \cdot v_i^2| \tag{5.37}$$

5.4.3. Cost function acceleration work

In general, if a constant force F acting along a distance s, the mechanical work W is performed by the force F on the distance s. Accordingly, the mechanical work W is the scalar product of the force F and the distance x (Stolz, 1995; Bartelmann et al., 2018).

$$W := \vec{F} \cdot \vec{x} = F \cdot \cos(\alpha) \cdot s = F_s \cdot s \tag{5.38}$$

Formula 5.38 assumes that the force F acts along the path s. Here F_s defines in Figure 5.7 the component of the force F that is in the path direction s (Rinner, 2018).

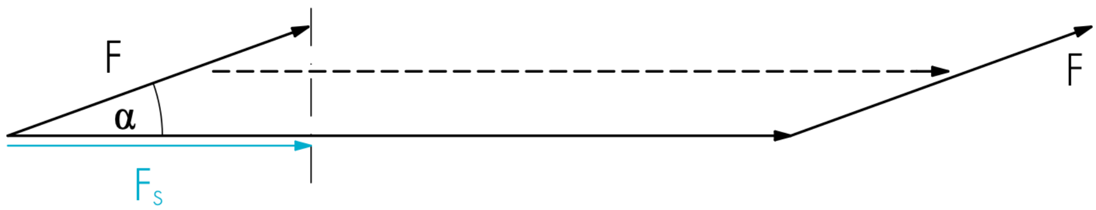

Figure 5.7.: Definition of mechanical work according to Rinner (2018)

Basically, the forms of mechanical work can be divided into acceleration work, stroke work and stress work. Acceleration work ∂W according to Formula 5.39 is the work done when a constant force F accelerates a body in the direction of the force F. If a force F acts on a body at rest, it is accelerated and performs acceleration work ∂W (Stolz, 1995; Bartelmann et al., 2018; Rinner, 2018)

$$\partial W = F \cdot \partial s \tag{5.39}$$

According to the 2nd Newtonian axiom the following applies to the force F

$$F = m \cdot a = \frac{\partial v}{\partial t} \tag{5.40}$$

and in general the following applies to the velocity v

$$\partial x = v \cdot \partial t \tag{5.41}$$

The equations 5.39 and 5.41 together result in

$$\partial W = m \cdot v \cdot \partial v \tag{5.42}$$

Integrating the Formula 5.42 provides the acceleration work to accelerate a mass m from rest to velocity v (Lüders and Pohl, 2017):

$$W = \frac{1}{2} \cdot m \cdot v^2 \tag{5.43}$$

In the context of this work, the velocity difference Δv_i is used for the velocity v in Formula 5.43, because otherwise there would be a continuous cost for the cost function acceleration. The velocity difference Δv_i in Formula 5.45 is calculated from the amount of the difference between the initial velocity v_i and the final velocity $v_{f,i}$ of the respective vehicle i. The final velocity $v_{f,i}$ can be calculated using Formula 5.44, where a_i is the acceleration of the vehicle i and t is the time horizon of the game.

$$v_{f,i} = a_i \cdot t + v_i \tag{5.44}$$

$$\Delta v_i = |v_i - v_{f,i}| \tag{5.45}$$

Based on the Formula 5.43 and Formula 5.45 the cost of acceleration work in this work for the individual vehicle i is calculated as follows:

$$J_{acc,i} = \frac{1}{2} \cdot m_i \cdot \Delta v_i^2 \tag{5.46}$$

5.5. Optimization of the cost function with the use of worst cases

As explained in Chapter 4.5, certain variables and parameters are optimized during optimization. In the following Chapters 5.5.1 to 5.5.6 the optimized cost functions of the Chapters 5.3 and 5.4 are explained.

5.5.1. Optimization of cost function collision energy

The worst case for the cost function collision energy can be defined as the largest speed difference between two vehicles. To obtain the largest possible speed difference between two vehicles according to Formula 5.17 and Formula 5.18, the maximum of the final speed of the two vehicles according to Formula 5.47 must be considered. Thus, the Formula of the speed difference according to Formula 5.16 is optimized as a function according to Chapter 4.5.

$$\max(\overline{\mathrm{V}}) = \max(\frac{1}{m_1 + m_2} \cdot \sqrt{(m_1 \cdot v_{1s})^2 + 2 \cdot m_1 \cdot m_2 \cdot v_{1s} \cdot v_{2s} \cdot \cos\alpha + (m_2 \cdot v_{2s})^2}) \tag{5.47}$$

5.5.2. Optimization of cost function Time-to-Collision

The worst case for the cost function TTC can be defined as the shortest distance between two vehicles. The lower the shortest distance $d_{TTC,s}$ between two vehicles, more lower is the TTC. As already explained in Chapter 5.3.2, a too low TTC entails not being able to react sufficiently to a hazardous situation (Lehsing, 2019; BMWI, 2020; Saffarzadeh et al., 2013). Based on this, the minimum of the Formula 5.21 is determined in the course of optimization according to Chapter 4.5. Moreover, since the cost of the cost function TTC according to Formula 5.22 and Formula 5.23 is speed dependent, the upper bound of the speed v_i of a vehicle i is considered as the optimized input of the cost function. The upper bound represents the maximum possible speed $\max(v_i)$ of the respective vehicle i, where the cost of the respective vehicle becomes larger with higher speed. Thus, the combination of the shortest distance $\min(d_{TTC,s})$ and the maximum possible speed $\max(v_i)$ represents the worst case for the cost function TTC, whose optimized cost $J_{opt,i}$ for the respective vehicle i can thus be determined according to Formula 5.48:

$$J_{opt,i} = \min(TTC_i) \cdot \max(v_i)^2 \tag{5.48}$$

5.5.3. Optimization of cost function center lane offset

The worst case can be defined for the cost function center lane offset from Chapter 5.3.3 can be defined as the largest distance between the respective vehicle i and the center lane. The larger the shortest distance $d_{clo,s}$ between the vehicle and the center lane, the larger the cost $J_{clo,i}$ provided that the shortest distance to the center lane $d_{clo,s}$ is larger than the tolerable distance to the center lane $d_{tol,clo}$ defined by Formula 5.24. Moreover, since the cost function center lane offset according to Formula and Formula is speed dependent, the upper bound of the speed v_i of a vehicle i is considered as the optimized input of the cost function. The upper bound represents the maximum possible speed $\max(v_i)$ of the respective vehicle i, where the cost of the respective vehicle becomes larger with higher speed. Thus, the combination of the largest shortest distance to the center lane $\max(d_{clo,s})$ and the highest speed $\max(v_i)$ represents the worst case for the cost function center lane offset, whose optimized cost $J_{opt,i}$ for the respective vehicle i can thus be determined according to Formula 5.49.

$$J_{opt,clo,i} = \max(d_{cl,s}) \cdot \max(v_i)^2 \tag{5.49}$$

5.5.4. Optimization of cost function Euclidean distance

The worst case can be defined for the cost function Euclidean distance between two vehicles from Chapter 5.3.4 as the shortest distance between two vehicles. The smaller the shortest distance $d_{eucl,s}$ between two vehicles, the larger the cost $J_{eucl,i}$. Moreover, since the cost of the cost function Euclidean distance according to Formula 5.27 is speed dependent, the upper bound of the speed v_i of a vehicle i is considered as an optimized input of the cost function. The upper bound represents the maximum possible speed $\max(v_i)$ of the respective vehicle i, where the cost of the respective vehicle becomes larger with higher speed. Thus, the combination of the shortest distance $d_{eucl,s}$ and the highest speed $\max(v_i)$ represents the worst case for the cost function euclidean distance, whose optimized cost $J_{opt,i}$ for the respective vehicle i can thus be determined according to Formula 5.50 and Formula 5.51:

$$J_{opt,eucl,i} = K_{low,i} \cdot \min(d_{eucl,s}) \cdot \max(v_i)^2 \tag{5.50}$$

$$J_{opt,eucl,i} = K_{up,i} \cdot \min(d_{eucl,s}) \cdot \max(v_i)^2 \tag{5.51}$$

5.5.5. Optimization of cost function obstacle distance

The worst case can be defined for the cost function obstacle distance from Chapter 5.3.5 can be defined as the shortest distance between a vehicle and an obstacle. The smaller the shortest distance $d_{eucl,s}$ between the vehicle and the obstacle, the larger the cost $J_{obs,i}$. Moreover, since the cost of the obstacle distance cost function according to Formula 5.30 is speed dependent, the upper bound on the speed v_i of a vehicle i is considered as the optimized input of the cost function. The upper bound represents the maximum possible speed $\max(v_i)$ of the respective vehicle i, where the cost of the respective vehicle becomes larger with higher speed. Thus, the combination of the shortest distance $d_{obs,s}$ and the highest speed $\max(v_i)$ represents the worst case for the cost function euclidean distance, whose optimized cost $J_{opt,i}$ for the respective vehicle i can thus be determined according to Formula 5.52.

$$J_{opt,obs,i} = \min(d_{obs,s}) \cdot \max(v_i)^2 \tag{5.52}$$

5.5.6. Optimization of the other relevant cost functions

When optimizing the other relevant cost functions from Chapter 5.4, which are also considered and optimized due to their safety relevance, no functions are optimized in contrast to the collision-avoiding cost functions. For these cost functions, only the input of the respective cost function is optimized. For the cost function quadratic velocity from Chapter 5.4.1 as well as for the cost function acceleration work from Chapter 5.4.3 the upper bound of the velocity is considered as optimized input, where the upper bound represents the maximum possible velocity of the respective vehicle. In the cost function quadratic velocity as well as in the cost function acceleration work the costs for the respective vehicle become larger with higher velocity. Thus, the upper bound of the speed for these two cost functions represents the worst case.

Also for the cost function steering angle from Chapter 5.4.2 the costs become larger with higher speed for the respective vehicle. In contrast to the cost functions quadratic velocity and acceleration work, the costs of the cost function steering angle also increase with a higher steering angle. Thus, only the combination of the upper bound of the velocity and the upper bound of the steering angle represents the worst case for the cost function steering angle.

5.6. Definition of combinations of cost functions

Within this Chapter, reasonable combinations are defined based on the previously defined cost functions. The cost functions of the quadratic speed, the steering angle and the acceleration from Chapter 5.4 are considered in each combination of cost functions and therefore not listed separately in the explanation of the cost function combination.

Since the cost function euclidean distance and the cost function TTC take into account the shortest distance between the two vehicles as the basis of their cost calculation, it does not make sense to simulate a combination of these cost functions. A combination of these two cost functions based on the shortest distance would only show the redundancy between the two functions. Therefore, reasonable cost calculations between the collision-avoiding cost functions are the following:

- cost function collision energy and cost function TTC
- cost function collision energy and cost function Euclidean distance
- cost function collision energy, cost function center lane offset and cost function TTC
- cost function collision energy, cost function Euclidean distance and cost function center lane offset

These listed combinations can be combined with the cost function obstacle distance to show on the basis of simulation results how the costs of the cost function combinations behave if they are triggered with an obstacle according to cost function obstacle from Chapter 5.3.5. The defined cost function combinations are also each combined with the cost functions for quadratic velocity, acceleration, and steering angle. In order to be able to weight the combinations of cost functions, factors are defined for the cost functions that are intended to avoid collisions. These are defined in the following list:

- weighting factor collision energy: $K_{l,ce}$
- weighting factor TTC: $K_{l,TTC}$
- weighting factor center lane offset: $K_{l,clo}$
- weighting factor euclidean distance: $K_{l,eucl}$
- weighting factor obstacle distance: $K_{l,obs}$

The factors for the cost functions, which do not directly avoid collisions, are defined in the following list:

- weighting factor quadratic velocity: $K_{l,vel}$
- weighting factor steering angle: $K_{l,steer}$
- weighting factor acceleration: $K_{l,acc}$

6. Simulation of the cost function in a normal form game

As explained before, reasonable cost functions as well as reasonable combinations of the cost functions shall be defined within the scope of this work. In order to check whether the cost functions from Chapter 5.3, which are intended to serve the avoidance of collisions, and from Chapter 5.4, which, however, do not directly serve the avoidance of collisions, must, however, be taken into account for reasons already explained, the cost functions are checked using the scenarios from Chapter 5.2.

The time horizon $t_{horizon}$ is set to 3.0 s according to Chapter 5.1. This time horizon $t_{horizon}$ has been chosen because within this time horizon all noteworthy results and trends of the cost evolution of a cost function can be represented. Moreover, the first simulation experiments have shown that a larger time horizon $t_{horizon}$ would increase the simulation time by a multiple. The parameters that generally apply to the lanes of the scenarios are listed in Table 6.1. Lane 1 represents the upper lane and lane 2 the lower lane in the scenarios from Chapter 5.2.

lane width	speed limit	y-position center lane 1	y-position center lane 2
3.5 m	27.7778 m/s	1.75	-1.75

Table 6.1.: Lane parameter

The uncertainties are defined in Chapter 5.1, as explained before. The effects of the uncertainties on the vehicle trajectory can be visualized by the Figure 6.1. For the vehicle on the upper lane in the left graph, uncertainties are not considered in the initial state of the vehicle as well as in the strategy of the vehicle. On the lower lane, however, uncertainties are considered in the initial state of the vehicle but no uncertainties are considered in the strategy of the vehicle. The left graph thus illustrates representative how uncertainty in the initial state affects the trajectory of the vehicle over a time interval of 3.0 s. When uncertainties in the vehicle's strategy are also considered in addition to the uncertainties in the initial state, it can be seen in the upper right graph that the range of the vehicle's possible trajectory becomes larger. To illustrate the effect of the speed at the same uncertainties, in the lower right graph a vehicle is simulated, which drives at a low speed at the same uncertainties in the initial state and in the strategy of the vehicle. The differences in the speeds show in the right graph that the higher the speed of a vehicle, the larger the range that the trajectory shows over a time interval of 3.0 s.

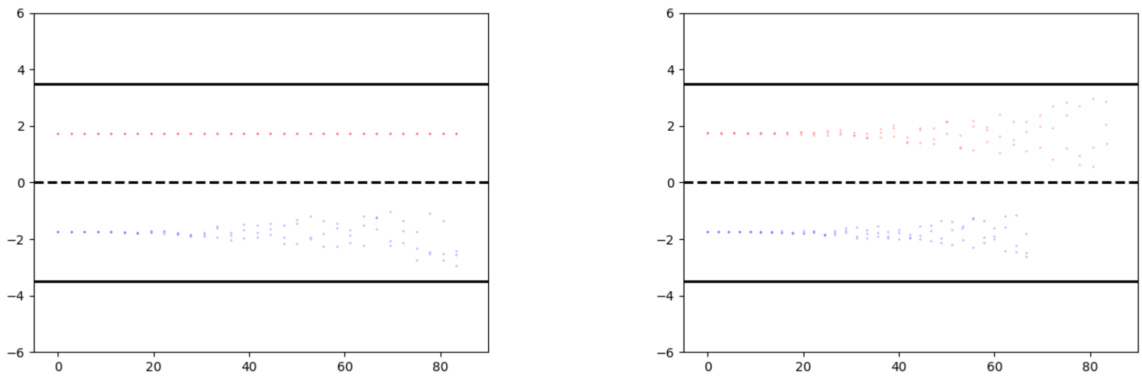

Figure 6.1.: Visualization of the effects of uncertainties on the vehicle trajectory

Concerning the strategy of acceleration according to Formula 5.3 in the simulation of cost functions it is distinguished in no acceleration and in acceleration of a free-time driver according to Schach et al. (2006) of 1.0 m/s. With the help of this distinction the effects and impacts of the strategy of acceleration in the simulation of the cost functions shall be clarified.

6.1. Simulation for single cost functions

Within this Chapter, the cost functions from Chapter 5.3 and Chapter 5.4 are simulated individually and evaluated and assessed with respect to the scenarios from Chapter 5.2. As can be seen from the results, it is difficult to interpret the calculated NE from the payoff matrices of the respective vehicles. The reason for this is that the uncertainties make interpretation difficult and the cost functions also show individual weaknesses. Therefore, an interpretation of the NE in the context of this work is not expedient and is therefore not considered further.

6.1.1. Simulation for cost function collision energy

In Table A.1 in Appendix A.1 several reasonable maneuvers were simulated for scenario 1. In each of these maneuvers, vehicle 1 changes back to its actual lane from which vehicle 1 started the overtaking maneuver. From the results, it is clear that regardless of the strategy, there is no cost for either vehicle 1 or vehicle 2. In the second scenario, a possible overtaking maneuver would not produce any significant results, since the two vehicles in scenario 2 are behind each other and would therefore not cause a collision, provided that the vehicles act rationally and no misbehavior by one of the two vehicles is to be expected.

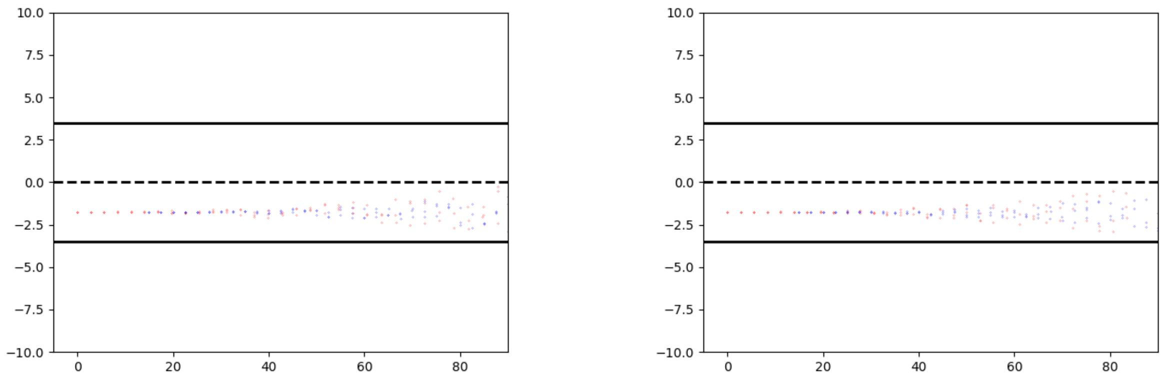

Figure 6.2.: Visualization of the maneuvers CE2.1 and CE2.2 of the first scenario in relation to the cost function collision energy

As shown in maneuver CE1.13 in Figure 6.2 on the left, the states of the two vehicles, which are shown as dots, do not overlap at a defined time $t_{horizon}$, so that, due to the head start of the vehicle 2 in front, the two vehicles cannot collide. If only vehicle 1 chooses the strategy to accelerate with 1 m/s (cf. maneuver CE2.2 in Figure 6.2), vehicle 1 overtakes vehicle 2 on the same lane in the defined time horizon $t_{horizon}$. Under real conditions, the two vehicles would cause a collision in this case and, consequently, costs would be incurred for this maneuver CE2.2 in scenario 2. However, as the Table A.2 shows, there are no costs for either vehicle in the second scenario. Calculated over a higher time horizon and without acceleration of vehicle 1 (cf. maneuver CE2.3 in Table A.2), the states vehicle 1 will also overtake vehicle 2, so that a potential collision is possible taking into account the uncertainties and strategies of the two vehicles and must lead to costs.

As can be seen in Table A.3, there is no cost to either vehicle for the reasonable maneuvers in scenario 3. As long as the vehicles continue to follow their respective lanes, they will not cause collisions, even when uncertainty is taken into account. Since it is possible under real conditions that both vehicles could start an overtaking maneuver of a third vehicle and carry it out, collisions are potentially possible, as visualized in the following Figure 6.3.

In the simulation results for the cost function collision energy in the Appendix A.1 it becomes clear that in all payoff matrices of the considered maneuvers of the three scenarios over the time

ID	u_veh1	u_veh2	J_veh1	J_veh2
CE1.1	[-0.004, 0.0], [-0.005, 0.0], [-0.006, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
CE1.2	[-0.007, 0.0], [-0.008, 0.0], [-0.009, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
CE1.3	[-0.004, 0.0], [-0.006, 0.0], [-0.008, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
CE1.4	[-0.004, 1.0], [-0.005, 1.0], [-0.006, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
CE1.5	[-0.007, 1.0], [-0.008, 1.0], [-0.009, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
CE1.6	[-0.004, 1.0], [-0.006, 1.0], [-0.008, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]

Table 6.2.: Simulation results of the second scenario for the cost function collision energy

horizon $t_{horizon}$ there are no costs for the two vehicles and thus according to the payoff matrices and the formulated cost function the two vehicles do not cause a collision. The reason for this is that the simulation only considers points as states for the vehicle and therefore a collision only occurs if the states overlap exactly. This is unlikely due to the different parameters, complexity and the defined uncertainties and is therefore reflected in the simulation results for the cost function collision energy. Due to the constant simulation results in Appendix A.1, the consideration of NE is not useful for the cost function collision energy and is therefore not taken into account. In Chapter 8 an approach is proposed, which addresses this problem and gives an outlook for future work.

In the following, we calculate as an example which costs would result for the defined maneuver CE2.3 from scenario 2 under real abstracted conditions. As explained before, under real conditions, costs must result from these maneuvers for both vehicles, provided that both vehicles choose the strategies from Formula 6.1 and Formula 6.2.

$$u_{veh1} = [[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]] \tag{6.1}$$

$$u_{veh2} = [[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]] \tag{6.2}$$

As the Figure 6.2 and Figure 6.3 illustrate, the first vehicle (red dots) overtakes the second vehicle (blue dots) at a higher speed when both vehicles follow their strategies over a defined

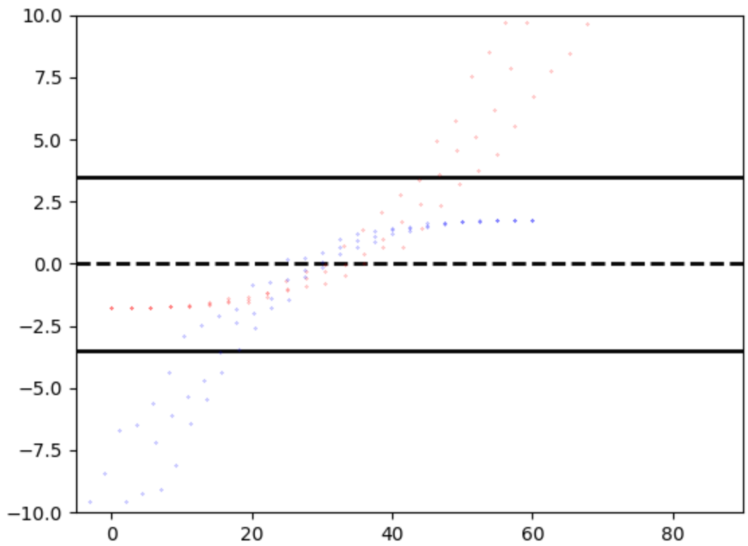

Figure 6.3.: Visualization of the simulation results of the maneuver CE3.12 of the third scenario in relation to the cost function collision energy

period of time. Considering the vehicle states and strategies, an accident would occur under real conditions. In the following, the actually expected simulation result for the previously explained maneuver of scenario 2 with its vehicle parameters is calculated. According to the Formula 5.16 from Chapter 5.3.1, the calculation of the final velocity $\overline{\mathrm{V}}$ at a certain time t with $m_1 = m_2 = 1000$ kg, $v_{1s} = 27.7778$ m/s, $v_{2s} = 25.0$ m/s and $\alpha = 0$ gives the following result:

$$\overline{\mathrm{V}} = \frac{1}{m_1 + m_2} \cdot \sqrt{(m_1 \cdot v_{1s})^2 + 2 \cdot m_1 \cdot m_2 \cdot v_{1s} \cdot v_{2s} \cdot \cos\alpha + (m_2 \cdot v_{2s})^2} = 26.3889 \text{ m/s} \tag{6.3}$$

As already explained in Formula 5.17 and Formula 5.18, the velocity change of the two vehicles $\overline{\triangle \mathrm{v}_1}$ and $\overline{\triangle \mathrm{v}_2}$ can be calculated as follows:

$$|\overline{\triangle \mathrm{V}_1}| = |\overline{\mathrm{V}} - \overline{\mathrm{v}_{1\mathrm{s}}}| = 1.3889 \text{ m/s} \tag{6.4}$$

$$|\overline{\triangle \mathrm{V}_2}| = |\overline{\mathrm{V}} - \overline{\mathrm{v}_{2\mathrm{s}}}| = 1,3889 \text{ m/s} \tag{6.5}$$

According to these calculations, the two $\overline{\triangle \mathrm{V}}$ values of the two vehicles should be in the range of 5 km/h < $\overline{\triangle \mathrm{V}}$ < 10 km/h and have the corresponding costs. Since this cost function is not meaningful in this form due to the unstated costs, it is necessary to combine this cost function with other cost functions in a meaningful way.

6.1.2. Simulation for cost function TTC

For the simulation of the cost function TTC in this work, a $d_{TTC,safe}$ of 3.4 m between the two vehicles in the initial condition was chosen. The basis for this definition of the parameter $d_{TTC,safe}$ is the distance between the two vehicles in scenario 1, which is 3.5 m. Since it is not reasonable to calculate the cost in the initial condition and since uncertainties in the initial condition have to be considered, the parameter $d_{TTC,safe}$ is defined accordingly. This is also reasonable for the simulation results in the initial state, since there is no cost for either vehicle in the initial state, since holds:

$$d_s > d_{TTC,safe} \tag{6.6}$$

If the defined safety distance $d_{TTC,safe}$ is not reached, both vehicles incur costs J_{TTC}, since the same safety distance $d_{TTC,safe}$ is defined for both vehicles. The simulation of the first scenario, as defined in Chapter 5.2, an overtaking maneuver that vehicle 1 wants to complete. For this purpose, vehicle 1 follows the strategy u_{veh1} to merge in front of vehicle 2 without incurring costs J_{TTC}. Since the costs are caused by falling below the safety distance, vehicle 1 must choose the strategy u_{veh1} depending on the strategy of vehicle 2 u_{veh2} in such a way that the safety distance $d_{TTC,safe}$ is not undercut. Since in scenario 1 only for vehicle 1 a strategy regarding the steering angle acceleration ω_u is a reasonable strategy, it is not considered for vehicle 2.

The simulation results of maneuver TTC1.1 and maneuver TTC1.2 in Table A.5 show that the larger the steering angle acceleration strategy ω_u, the larger the cost J_{TTC} becomes for the two vehicles. Based on Figure 6.4, this can be understood as vehicle 1 approaches vehicle 2 over the defined time horizon $t_{horizon}$ faster with a high strategy of steering angle acceleration ω_u than with a small strategy chosen. Since the cost function according to Formula 5.22 as well as Formula 5.23 is speed-dependent and the two vehicles complete their driving task at different speeds, the costs of the cost function TTC are calculated individually for the two vehicles and are therefore not identical for scenario 1.

Provided that in the strategy of vehicle 1 the strategy of acceleration a_u with 1 m/s is considered, already for lower strategies in the steering angle acceleration ω_u costs arise for both vehicles, since the vehicle reaches a higher speed over the defined time horizon $t_{horizon}$ due to the constant acceleration and thus the shortest distance d_s between the two vehicles exceeds the safety distance

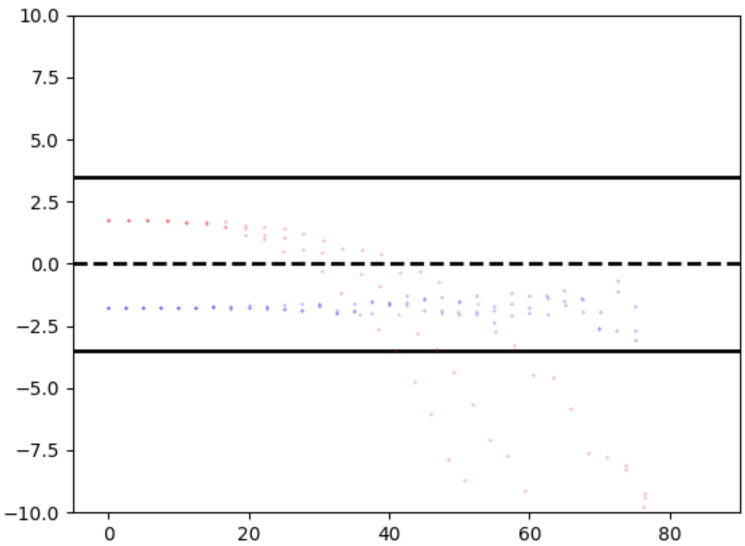

Figure 6.4.: Visualization of the simulation results of the maneuver TTC1.1 of the first scenario in relation to the cost function TTC

$d_{TTC,safe}$ for lower strategies in the steering angle acceleration ω_u (cf. TTC1.3 and TTC1.4 in Table A.5). Similar effects are observed for the acceleration of vehicle 2 (cf. TTC1.5 and TTC1.6 in Table A.5). Due to the higher speed of vehicle 2 caused by the constant acceleration, the shortest distance d_s between the two vehicles falls below the safety distance $d_{TTC,safe}$ at an earlier time t, consequently the cost for the two vehicles already increases for smaller strategies in the steering angle acceleration ω_u. Another effect shown by the simulation results in Table A.5 that the costs for the two vehicles become smaller again at a certain increasing steering angle acceleration ω_u. The reason for this is that the steering angle acceleration is so large from a certain value that the shortest distance d_s between the two vehicles within the defined time horizon $t_{horizon}$ becomes larger than the safety distance $d_{TTC,safe}$ again from a certain point in time t and therefore no more costs are generated, which reduces the cost value for the respective strategy.

Reasonable maneuvers of the second scenario, which have to be considered for the evaluation of the cost function TTC and on which the results of the simulation can be explained, are only formed considering the strategy of acceleration a_u of vehicle 1. The reason for this is that without considering the strategy of acceleration a_u for vehicle 1, no costs J_{TTC} arise for the two vehicles, since the safety distance $d_{TTC,safe}$ is not undercut for the defined time horizon $t_{horizon}$ (cf. TTC2.1 in Table 6.3). However, as long as both vehicles follow the strategy of accelerating with $a_u = 1m/s$, there is again no cost for either vehicle, since the shortest distance d_s between both vehicles does not fall below the safety distance $d_{TTC,safe}$ in the defined time horizon $t_{horizon}$ (cf. TTC2.3 in Table 6.3).

ID	u_veh1	u_veh2	J_veh1	J_veh2
TTC1.1	[-0.01, 0.0], [-0.02, 0.0], [-0.03, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [3623.84 3610.18 4555.80]	[-0. -0. -0.] [-0. -0. -0.] [2935.34 2924.28 3690.27]
TTC1.2	[-0.03, 0.0], [-0.04, 0.0], [-0.05, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[2706.69 2728.59 3666.63] [6086.37 5116.13 5125.57] [5827.68 5784.06 5803.04]	[2192.56 2210.24 2969.99] [4930.09 4144.11 4151.59] [4720.57 4685.17 4700.50]
TTC1.3	[-0.01, 1.0], [-0.02, 1.0], [-0.03, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
TTC1.4	[-0.03, 1.0], [-0.04, 1.0], [-0.05, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [2266.36 1477.64 1479.80] [3534.11 3550.76 3545.93]	[-0. -0. -0.] [1726.87 1119.03 1120.78] [2686.82 2699.70 2696.01]
TTC1.5	[-0.01, 0.0], [-0.02, 0.0], [-0.03, 0.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[9743.11 11877.12 7336.70] [14767.54 12252.47 14743.60] [12225.71 10917.65 10931.95]	[9033.56 10988.68 6809.87] [13280.19 10962.39 13255.40] [10815.65 9693.76 9704.63]
TTC1.6	[-0.03, 0.0], [-0.04, 0.0], [-0.05, 0.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[12118.10 12181.01 12260.79] [9261.10 11090.78 9878.72] [9025.62 9030.16 9077.01]	[10718.61 10774.26 10848.95] [8082.54 9733.51 8704.09] [7884.91 7887.14 7929.43]
TTC1.7	[-0.01, 1.0], [-0.02, 1.0], [-0.03, 1.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-0. -0. -0.] [-0. -0. -0.] [4860.41 3891.81 4887.46]	[-0. -0. -0.] [-0. -0. -0.] [3970.35 3180.33 3991.76]
TTC1.8	[-0.03, 1.0], [-0.04, 1.0], [-0.05, 1.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[4828.85 4836.59 4816.23] [6451.62 6449.83 6412.25] [6076.13 6064.63 6049.58]	[3944.58 3950.92 3934.29] [5266.22 5264.77 5234.21] [4955.73 4946.41 4934.14]

Table 6.3.: Simulation results of the second scenario for the cost function TTC

As the Table 6.3 in TTC2.4 and TTC2.5 illustrates, only for a strategy of steering angle acceleration $\omega_u >= 0.002$ there is no cost for the two vehicles. Approximately from this strategy value of steering angle acceleration ω_u the shortest distance d_s between the two vehicles is larger than safety distance $d_{TTC,safe}$ thus satisfying the Formula 6.6. Within the scope of this work, an exact strategy value for the steering angle acceleration is not determined.

The third scenario, as described in Chapter 5.2, illustrates the situation where two vehicles are supposed to pass each other on different lanes. However, if certain strategy of steering angle acceleration ω_u of vehicle 1 is taken into account, there is a cost for the two vehicles that the shortest distance d_s falls below the safety distance $d_{TTC,safe}$ (cf. TTC3.2, TTC3.3 and TTC3.4 in Table A.6). Looking at the calculated costs of these maneuvers, it is noticeable that the costs for the two vehicles start at a certain strategy of steering angle acceleration ω_u, increase and then decrease again until no costs are calculated again. The reason for this effect can be explained with the Figure 6.5 based on TTC3.16.

From a certain value of the steering angle acceleration strategy ω_u, which is not defined in more detail in this work, the steering angle acceleration is so large that vehicle 1 has intersected the lane of vehicle 2 before vehicle 2 passes the same point on the lane or the safety distance $d_{TTC,safe}$ can

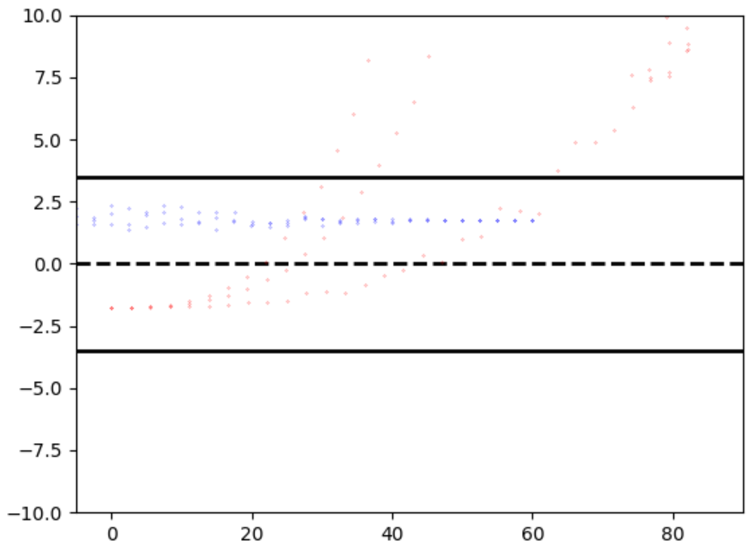

Figure 6.5.: Visualization of the simulation results of the maneuver TTC3.16 of the third scenario in relation to the cost function TTC

be undercut. The situation is similar with a very small strategy of the steering angle acceleration ω_u. If this is very small, vehicle 1 intersects the trajectory of vehicle 2 at a time t in the time horizon $t_{horizon}$ at which vehicle 2 has already reached a distance to this point that is above the safety distance $d_{TTC,safe}$, resulting in no costs for both vehicles.

The effects already explained for the cost function TTC can also be applied to other strategy combinations within the scenarios. The simulation results are listed in the Appendix A.2.

6.1.3. Simulation for cost function center lane offset

As described in Chapter 5.3.3, the costs are influenced by the factor $K_{twt,cl}$ for two way traffic. In the context of this work, only two way traffic is considered in the scenarios according to Chapter 5.2, so this factor can be excluded from the evaluation and interpretation of the simulation results and thus this parameter does not need to be considered for the cost of the cost function of the center lane offset. However, in future work this factor can be taken into account, provided that different road types are considered in the scenarios of the simulation. Another factor that needs to be defined as part of the simulation of the cost function center lane offset is the factor $K_{tol,clo}$. The factor $K_{tol,clo}$ is defined as 1/8 in the context of this work, so that a defined lane width of 3.5 m results in a tolerance range of 0.4375 m to the left and to the right of the lane center.

In the first scenario, vehicle 1 is in an overtaking maneuver of vehicle 2, as described in Chapter 5.2. Accordingly, there is an enormous cost for vehicle 1 if it does not complete the overtaking maneuver (cf. CLO1.1 in Table 6.4). Vehicle 2, on the other hand, is on the designated lane in the first scenario and therefore there is only a small cost for CLO1.1. The small cost arises because vehicle 2 leaves the defined acceptable tolerance range due to the defined uncertainties at a certain time t in the time horizon $t_{horizon}$, resulting in a cost J_{clo}. Provided that for the vehicle 2 the strategy of acceleration a_u is considered, these costs increase, because the vehicle reaches a higher speed due to the constant acceleration and thus the costs according to Formula 5.5.3 are higher for the vehicle.

ID	u_veh1	u_veh2	J_veh1	J_veh2
CLO1.1	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-83016.86 -83016.86 -83016.86] [-82633.20 -82633.20 -82633.20] [-81780.68 -81780.68 -81780.68]	[-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96]
CLO1.2	[-0.01, 0.0], [-0.03, 0.0], [-0.05, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-84124.55 -84124.55 -84124.55] [-216615.02 -216615.02 -216615.02] [-312745.46 -312745.46 -312745.46]	[-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96]
CLO1.3	[-0.001, 0.0], [-0.01, 0.0], [-0.03, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-75992.56 -75992.56 -75992.56] [-82324.18 -82324.18 -82324.18] [-216809.78 -216809.78 -216809.78]	[-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96]
CLO1.4	[-0.008, 0.0], [-0.009, 0.0], [-0.01, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-72929.92 -72929.92 -72929.92] [-77150.33 -77150.33 -77150.33] [-83906.62 -83906.62 -83906.62]	[-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96]
CLO1.5	[-0.004, 0.0], [-0.005, 0.0], [-0.006, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-56511.21 -56511.21 -56511.21] [-60167.62 -60167.62 -60167.62] [-63132.50 -63132.50 -63132.50]	[-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96]
CLO1.6	[-0.01, 1.0], [-0.03, 1.0], [-0.05, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-101582.38 -101582.38 -101582.38] [-282376.29 -282376.29 -282376.29] [-394580.40 -394580.40 -394580.40]	[-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96]
CLO1.7	[-0.001, 1.0], [-0.01, 1.0], [-0.03, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-80004.47 -80004.47 -80004.47] [-105844.15 -105844.15 -105844.15] [-282205.97 -282205.97 -282205.97]	[-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96]
CLO1.8	[-0.008, 1.0], [-0.009, 1.0], [-0.01, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-86605.31 -86605.31 -86605.31] [-94833.96 -94833.96 -94833.96] [-104465.37 -104465.37 -104465.37]	[-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96]
CLO1.9	[-0.004, 1.0], [-0.005, 1.0], [-0.006, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-62608.19 -62608.19 -62608.19] [-70613.70 -70613.70 -70613.70] [-71268.80 -71268.80 -71268.80]	[-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96]
CLO1.10	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-95495.92 -95495.92 -95495.92] [-89779.41 -89779.41 -89779.41] [-87747.39 -87747.39 -87747.39]	[-4312.18 -5212.02 -4633.39] [-4312.18 -5212.02 -4633.39] [-4312.18 -5212.02 -4633.39]

Table 6.4.: Simulation results of the first scenario for the cost function center lane offset

In general, based on the results from Table 6.4 for vehicle 1 in scenario 1, it can be concluded that the lower the strategy of steering angle acceleration ω_u is chosen, the lower the costs are for vehicle 1. These results cannot be considered reasonable, since a slow approach to the lane should cause higher costs than a fast approach. In the case of the simulation, this can be justified by the fact that by choosing a strategy, vehicle 1 does not stop the simulation after reaching the center of the lane, but follows the strategy over the entire time horizon $t_{horizon}$, thus driving beyond the center of the lane.

The simulation results in Table A.8 for vehicle 2 in the second scenario can be interpreted in the same way as in the first scenario, that vehicle 2 does not move away from the lane center with respect to the sensible maneuvers, but only the strategy of acceleration a_u is considered. The first vehicle, on the other hand, starts the overtaking maneuver, which causes the vehicle to move away from the center of the lane. In this case, based on the simulation results in Table A.8, the statement can be made that the higher the strategy of steering angle acceleration ω_u is chosen, the greater the cost for vehicle 1. The reason for this is that the defined tolerance range is exceeded at an earlier point t with a larger steering angle acceleration strategy ω_u and is therefore also further away from the limit of the tolerance range at a later point in time than lower steering angle acceleration strategies ω_u.

The effects already explained for the cost function center lane offset can also be applied to other strategy combinations within scenarios 1 and 2 as well as to scenario 3. The simulation results for this are shown in Appendix A.3.

6.1.4. Simulation for cost function Euclidean distance

For the Euclidean distance cost function simulation in this work, a safety distance $d_{eucl,safe}$ of 3.4 m between the two vehicles in the initial condition is chosen. The basis for this definition of the parameter $d_{eucl,safe}$ is the distance between the two vehicles in scenario 1, which is 3.5 m. Since it is not reasonable to calculate the cost in the initial condition and since uncertainties in the initial condition must be taken into account, the parameter $d_{eucl,safe}$ is defined accordingly. This is also reasonable for the simulation results in the initial state, since there is no cost for either vehicle in the initial state, since holds:

$$d_s > d_{eucl,safe} \tag{6.7}$$

As long as the defined safety distance $d_{eucl,safe}$ is not reached, costs J_{eucl} result for both vehicles, since the same safety distance $d_{eucl,safe}$ is defined for both vehicles. Since the cost function Euclidean distance, as explained in Chapter 5.3.4, only determines the cost for vehicles traveling on the same roadway, there is no cost for the two vehicles for both the first scenario and the third scenario, since different roadways were defined for the two vehicles in Chapter 5.2. For this reason, the attached simulation results in Table A.10 and Table A.12 in Appendix A.4.

Reasonable maneuvers of the second scenario, which have to be considered for the evaluation of the cost function Euclidean distance and on which the results of the simulation can be explained, are only formed considering the strategy of acceleration a_u of vehicle 1. The reason for this is that without considering the acceleration for vehicle 1, there are no costs for the two vehicles, since the safety distance $d_{eucl,safe}$ is not undercut for the defined time horizon $t_{horizon}$ (cf. ED2.1 in Table A.11). However, if both vehicles follow the strategy to accelerate with $a_u = 1$ m/s, again no costs arise for the two vehicles, since the shortest distance between both vehicles does not fall below the safety distance $d_{eucl,safe}$ for the defined time horizon $t_{horizon}$ (cf. ED2.3 in Table A.11).

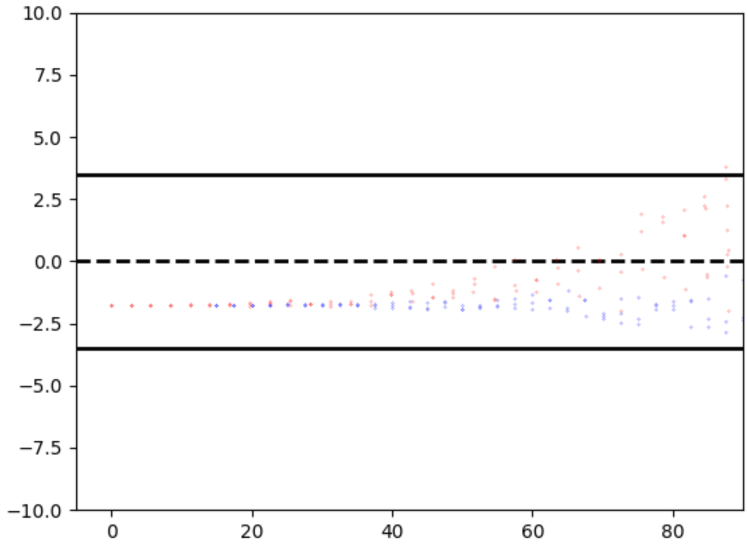

Figure 6.6.: Visualization of the simulation results of the maneuver ED2.5 of the second scenario in relation to the cost function Euclidean distance

As the results of the Table A.11 in ED2.4 and ED2.5 output and the Figure 6.6 visualizes, only for a strategy of steering angle acceleration $\omega_u >= 0.002$ there is no cost for the two vehicles. Approximately from this strategy value of steering angle acceleration ω_u the shortest distance d_s between the two vehicles is larger than safety distance $d_{eucl,safe}$ and thus the Formula 6.7 is satisfied. Within the scope of this work, an exact strategy value for the steering angle acceleration is not determined.

6.1.5. Simulation for cost function obstacle distance

For the cost function obstacle distance according to Chapter 5.3.5, the following obstacle state is defined:

$$obstacle = [58, 59], [-1.75, -1.75] \tag{6.8}$$

Due to the implemented obstacle in the scenarios from Chapter 5.2 the reasonable maneuvers in the scenarios change for the cost function obstacle distance, because otherwise the respective vehicle would collide with the obstacle. Costs J_{obs} for the respective vehicle only arise if the shortest distance between the respective vehicle and the obstacle $d_{obs,s}$ is smaller than the defined safety distance $d_{obs,safe}$ of 3.4 m.

$$d_{obs,s} > d_{obs,safe} \tag{6.9}$$

As the Figure 6.7 illustrates, the maneuver for the first scenario changes in that only vehicle 2 will pursue the intention to change lanes, as it would otherwise collide with the obstacle. As the simulation results in Table A.13 in Appendix A.5 show, there is a cost to vehicle 1 if this vehicle does not pursue a strategy. This cost arises from considering the uncertainties for Vehicle 1, which can be seen in Figure 6.7, and increases when considering the strategy of acceleration a_u over the defined time interval, since the shortest distance to the obstacle falls below the safety distance due to a constant acceleration within the defined time interval at an earlier time (cf. OBS1.1 and OBS1.2 in Table A.13). As can be seen from Table A.13 in maneuvers OB1.3 and OBS1.4, no costs are generated for vehicle 2 only when a certain steering angle acceleration ω_u is reached. An exact threshold value, above which no costs are generated for vehicle 2, is not determined within the scope of this work. Provided that for the strategy of vehicle 2 the acceleration a_u is considered in addition to the steering angle acceleration ω_u, it becomes clear from the maneuvers OBS1.5 and OBS1.6 that the costs for vehicle 2 increase. This is understandable since, for a constant acceleration of vehicle 2, the shortest distance to the obstacle $d_{obs,s}$ at an earlier time is smaller than the safety distance to the obstacle $d_{obs,safe}$ and, therefore, higher costs J_{obs} are incurred.

For the second scenario, the maneuvers change in that not only vehicle 1 starts to overtake vehicle 2, but also vehicle 2 wants to pass the obstacle with the goal of generating as few costs as possible. As can be seen from Table A.15 in the maneuvers OBS2.1 and OBS2.3, no costs are generated for the front vehicle 2 only if a certain steering angle acceleration ω_u is reached. An exact limit value above which no more costs are generated for vehicle 2 is not determined in the context of this work. If for the strategy of vehicle 2 the acceleration a_u is considered in addition to the steering angle acceleration ω_u, it becomes clear from the maneuvers OBS2.2 and OBS2.4 that

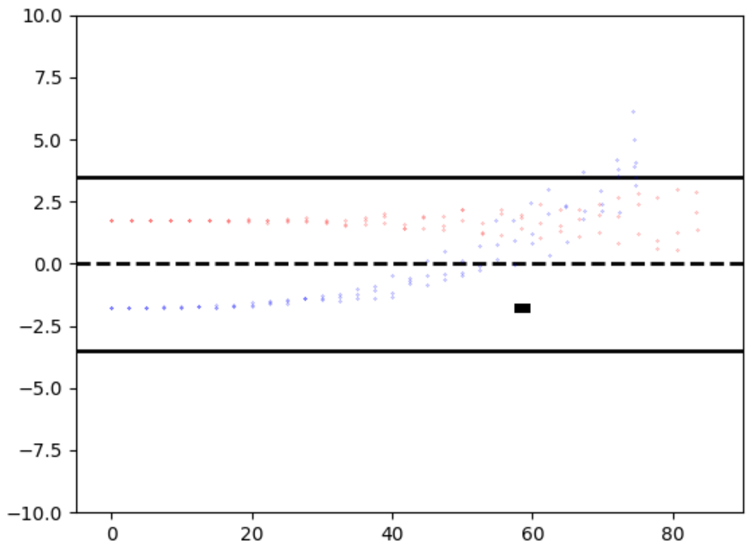

Figure 6.7.: Visualization of the simulation results of the maneuver OBS1.4 of the first scenario in relation to the cost function obstacle distance

the costs for vehicle 2 increase. This is understandable since, for a constant acceleration of vehicle 2, the shortest distance to the obstacle $d_{obs,s}$ at an earlier time is smaller than the safety distance to the obstacle $d_{obs,safe}$ and therefore higher costs J_{obs} are incurred.

ID	u_veh1	u_veh2	J_veh1	J_veh2
OBS2.1	[0.005, 0.0], [0.007, 0.0], [0.009, 0.0]	[0.015, 0.0], [0.016, 0.0], [0.017, 0.0]	[4509.06 4509.06 4509.06] [-0. -0. -0.] [-0. -0. -0.]	[2120.73 2118.73 -0.] [2120.73 2118.73 -0.] [2120.73 2118.73 -0.]
OBS2.2	[0.005, 0.0], [0.007, 0.0], [0.009, 0.0]	[0.015, 1.0], [0.016, 1.0], [0.017, 1.0]	[4509.06 4509.06 4509.06] [-0. -0. -0.] [-0. -0. -0.]	[4640.10 2369.82 -0.] [4640.10 2369.82 -0.] [4640.10 2369.82 -0.]
OBS2.3	[0.005, 1.0], [0.007, 1.0], [0.009, 1.0]	[0.015, 0.0], [0.016, 0.0], [0.017, 0.0]	[1897.22 1897.22 1897.22] [2709.46 2709.46 2709.46] [-0. -0. -0.]	[2136.44 -0. -0.] [2136.44 -0. -0.] [2136.44-0. -0.]
OBS2.4	[0.005, 1.0], [0.007, 1.0], [0.009, 1.0]	[0.015, 1.0], [0.016, 1.0], [0.017, 1.0]	[1897.22 1897.22 1897.22] [2709.46 2709.46 2709.46] [-0. -0. -0.]	[2415.00 2451.41 -0.] [2415.00 2451.41 -0.] [2415.00 2451.41 -0.]

Table 6.5.: Simulation results of the second scenario for the cost function obstacle distance

The same effects can be seen for the following vehicle 1. In Table 6.5, even for a given strategy of steering angle acceleration ω_u, there is no cost J_{obs} for vehicle 1, since the shortest distance to the obstacle $d_{obs,s}$ exceeds the safety distance to the obstacle $d_{obs,safe}$. Also, for the consideration of acceleration a_u, from the comparison with the maneuvers without considering acceleration, the cost for vehicle 1 becomes lower than the safety distance at an earlier time when the acceleration of vehicle 1 is constant, and therefore the cost is larger in the defined time horizon.

For the third scenario, the maneuvers change in that vehicle 1 wants to overtake the obstacle because it does not want to collide with the obstacle and does not want to generate a cost. As can be seen from Table A.15 in maneuvers OBS3.6 and OBS3.7, no costs are generated for vehicle 1 only when a certain steering angle acceleration ω_u is reached. An exact threshold value, above which no costs are generated for vehicle 1, is not determined within the scope of this work. Provided that for the strategy of vehicle 1 the acceleration a_u is considered in addition to the steering angle acceleration ω_u, it becomes clear from the maneuvers OBS3.4 and OBS3.5 that the costs for vehicle 1 increase. This is understandable since, for a constant acceleration of vehicle 1, the shortest distance to the obstacle $d_{obs,s}$ at an earlier time is smaller than the safety distance to the obstacle $d_{obs,safe}$ and therefore higher costs J_{obs} are incurred. There is no cost for vehicle 2 because it does not fall below the safety distance (cf. OBS3.1 in Table A.15). Also considering the acceleration (cf. OBS3.2 in Table A.15) as well as considering a large strategy value of the steering angle acceleration ω_u (cf. OBS3.3 in Table A.15), there are no costs J_{obs} for vehicle 1, since the safety distance is not undercut.

6.1.6. Simulation for cost function quadratic velocity

For the cost function quadratic velocity, the following general conditions are defined in this work. The speed limit tolerance v_T, which was already explained in Chapter 5.4.1, is defined as 1 m/s, so that costs are only incurred for the respective vehicle if the deviation of the speed of the respective vehicle from the speed limit is greater than the speed limit tolerance range. With a defined speed limit v_L of 27.7778 m/s, costs are only calculated for the respective vehicle for speeds outside the tolerance range of 26.7778 m/s to 28.7778 m/s. As explained in Chapter 5.4.1, for a low speed than the defined tolerance range, the factor K_{vel} was introduced to be able to weight the cost $J_{vel,i}$ lower for a low speed. In the context of the simulation, this factor K_{vel} is defined as 0.1, so that the effects of the differences can be clearly identified and evaluated from the results.

In the first scenario there are no costs for vehicle 1, because the speed of vehicle 1 is defined with a speed of 27.7778 m/s and thus corresponds exactly to the speed limit v_L. Due to the defined tolerance range, the considered uncertainties can be absorbed for the defined time horizon $t_{horizon}$ = 3.0 s, so that no costs arise for vehicle 1 due to the consideration of uncertainties. Vehicle 2 is outside the tolerance range with a velocity $v_{veh2} = 25.0$ m/s, so that costs arise for this vehicle. These conclusions are supported by the results for scenario 1 in Table 6.6 in Appendix A.6. Furthermore, these results show that the cost $J_{vel,i}$ does not change with the steering angle acceleration strategy ω_u. This result is to be expected since, according to Chapter 5.4.1, the steering angle acceleration ω cannot affect the cost function quadratic velocity. As the following Table 6.6 shows, a change in acceleration a, and therefore choosing a strategy a_u that includes a value for acceleration a, affects the payoff matrices of the two vehicles. The larger the acceleration of the vehicle, the larger the cost of that vehicle will be, since acceleration logically affects speed. Since the selected strategy for acceleration is a constant acceleration over the defined time horizon, the cost becomes larger over a larger time horizon $t_{horizon}$, as VQ1.10 in Table 6.6 shows.

ID	u_veh1	u_veh2	J_veh1	J_veh2
VQ1.1	[0.0, 0.0], [0.0, 0.0], [0.00, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-9.79 -9.79 -9.79] [-9.79 -9.79 -9.79] [-9.79 -9.79 -9.79]
VQ1.2	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-9.79 -9.79 -9.79] [-9.79 -9.79 -9.79] [-9.79 -9.79 -9.79]
VQ1.3	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-9.79 -9.79 -9.79] [-9.79 -9.79 -9.79] [-9.79 -9.79 -9.79]
VQ1.4	[0.07, 0.0], [0.08, 0.0], [0.09, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-9.79 -9.79 -9.79] [-9.79 -9.79 -9.79] [-9.79 -9.79 -9.79]
VQ1.5	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-9.79 -9.79 -9.79] [-9.79 -9.79 -9.79] [-9.79 -9.79 -9.79]
VQ1.6	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-9.79 -9.79 -9.79] [-9.79 -9.79 -9.79] [-9.79 -9.79 -9.79]
VQ1.7	[0.07, 0.0], [0.08, 0.0], [0.09, 0.0]	[0.07, 0.0], [0.08, 0.0], [0.09, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-9.79 -9.79 -9.79] [-9.79 -9.79 -9.79] [-9.79 -9.79 -9.79]
VQ1.8	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-9.79 -9.79 -9.79] [-9.79 -9.79 -9.79] [-9.79 -9.79 -9.79]
VQ1.9	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[-28.70 -28.70 -28.70] [-0.96 -0.96 -0.96] [-0. -0. -0.]	[-2.03 -3.89 -9.79] [-2.03 -3.89 -9.79] [-2.03 -3.89 -9.79]
VQ1.10	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[-94.55 -94.55 -94.55] [-7.17 -7.17 -7.17] [-0. -0. -0.]	[-2.09 -3.90 -12.95] [-2.09 -3.90 -12.95] [-2.09 -3.90 -12.95]
VQ1.11	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-97.96 -97.96 -97.96] [-97.97 -97.97 -97.97] [-97.97 -97.97 -97.97]	[-0.46 -0.46 -0.46] [-0.46 -0.46 -0.46] [-0.46 -0.46 -0.46]
VQ1.12	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0.46 -0.46 -0.46] [-0.46 -0.46 -0.46] [-0.46 -0.46 -0.46]	[-64.33 -64.33 -64.33] [-64.33 -64.33 -64.33] [-64.33 -64.33 -64.33]

Table 6.6.: Simulation results of the first scenario for the cost function quadratic velocity

By squaring the speed difference according to Formula 5.35 and Formula 5.36, stronger deviations result in larger costs (see VQ1.12). However, a difference in the deviation in velocity becomes apparent when the two results for the two vehicles in VQ1.11 are compared. In this maneuver the speeds of vehicle 1 were adjusted to 30.5556 m/s and of vehicle 2 to 26.3889 m/s and thus the speed difference for both vehicles to the speed limit v_L is 1.38889 m/s, whose value is above the tolerance limit of 1 m/s and thus causes costs for both vehicles. Based on the cost calculation according to Formula 5.35, the same costs should be calculated for both vehicles, since it is the same speed difference. In this case, however, the factor for undercutting the speed limit v_L comes into effect, which means that only about 10 percent of the costs according to Formula 5.36 can be calculated for the slower vehicle 2 by the factor of 0.1, taking into account the uncertainties.

Since the strategy of angular acceleration ω_u does not affect the cost function quadratic velocity and the angular acceleration has decisive changes in the scenarios due to the maneuvers, the same results have been simulated in scenarios 2 and 3 and its maneuvers. Therefore, the Table A.17 and Table A.18 in Appendix A.6 show the same results as Table 6.6, which can therefore also be interpreted in exactly the same way.

6.1.7. Simulation for cost function steering angle

Based on the definition of the cost function steering angle from Chapter 5.4.2 in which the dependence of the costs according to Formula 5.37 on the steering angle and speed is defined, individual results are to be expected for the simulation results both when changing the strategy of the steering angle acceleration ω_u and when changing the strategy of the acceleration a_u. In the following Table 6.7 excerpt from Appendix A.7 for the first scenario, when comparing the results for the ST1.1 maneuver, it is clear that even with a strategy of no steering angular velocity ω_u, there is a cost for the respective vehicles. The reason for this is the consideration of uncertainties in the strategy and initial state for the steering angle as defined in Chapter 5.1. Since the two vehicles drive at different speeds, different results are obtained due to the dependence of the cost function steering angle on the speed according to Formula 5.37. The higher the speed of the vehicle, the higher the cost of the respective vehicle. Considering the strategy with acceleration a_u, among others, it becomes clear in Maneuver ST1.2 in Table 6.7 that acceleration has an impact on the cost of the cost function steering angle. This can be justified by the fact that the constant acceleration contributes to the fact that the speed of the vehicles increases. Due to the speed dependence of the cost function according to the Formula 5.37, the costs for the respective vehicle increase under consideration of the acceleration strategy.

Considering a strategy regarding the steering angle acceleration ω_u for vehicle 1 in scenario, it can be concluded based on the results for maneuver ST1.3 and maneuver ST1.4 from Table 6.7 that the larger the strategy of the steering angle acceleration ω_u, the larger the cost for the respective vehicle. From this result was expected based on the defined Formula 5.37 for the cost of the cost function steering angle. The simulation results in Appendix A.7 for the reasonable maneuvers in Scenario 2 (cf. Table A.20) and Scenario 3 (cf. Table A.21) support the results explained earlier.

ID	u_veh1	u_veh2	J_veh1	J_veh2
ST1.1	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-10.52 -10.52 -10.52] [-13.32 -13.32 -13.32] [-14.51 -14.51 -14.51]	[-9.76 -10.44 -11.29] [-9.76 -10.44 -11.29] [-9.76 -10.44 -11.29]
ST1.2	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-15.03 -15.03 -15.03] [-13.47 -13.47 -13.47] [-12.50 -12.50 -12.50]	[-11.35 -10.82 -7.86] [-11.35 -10.82 -7.86] [-11.35 -10.82 -7.86]
ST1.3	[-0.01, 0.0], [-0.03, 0.0], [-0.05, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-360.59 -360.59 -360.59] [-1078.60 -1078.60 -1078.60] [-1793.01 -1793.01 -1793.01]	[-10.18 -9.97 -10.75] [-10.18 -9.97 -10.75] [-10.18 -9.97 -10.75]
ST1.4	[-0.001, 0.0], [-0.005, 0.0], [-0.009, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-35.30 -35.30963221 -35.30] [-176.84 -176.84577504 -176.84] [-323.46 -323.46 -323.46]	[-10.31 -10.05 -10.22] [-10.31 -10.05 -10.22] [-10.31 -10.05 -10.22]

Table 6.7.: Excerpt of the simulation results of the first scenario for the cost function steering angle from Table A.19 from Appendix A.7

6.1.8. Simulation for cost function acceleration work

The simulation of the cost function acceleration from Chapter 5.4.3 is also similar to the simulation of the cost function quadratic velocity, since the strategy of the steering angle acceleration ω_u has no effect on the cost function and therefore no costs can arise for the respective vehicle when choosing a strategy with respect to the steering angle acceleration ω. The results for the first scenario AC1.1 to AC1.4 in Table A.22 show the simulation results for the cost function acceleration with different strategies for the steering angle acceleration for the two vehicles.

The Formula 5.46 from Chapter 5.4.3 shows that only with changes in the strategy for the acceleration a_u in rad/s or a change in speed can costs arise for the respective vehicle. Maneuvers AC1.5 to AC1.7 of the first scenario in Table A.22 show how the costs for the respective vehicles behave for different strategies for acceleration a_u. The results for AC1.5 to AC1.7 show that the larger the strategy for acceleration a_u chosen by a vehicle, the larger the cost $J_{acc,i}$ for this chosen strategy. When the simulation results of AC1.7 are compared with the results of AC1.8, whose strategies were simulated over a larger time horizon of $t_{horizon} = 4.0$ s, it can be seen that the cost $J_{acc,i}$ also increases over a larger time horizon. This can be explained by the fact that the strategy is a constant one, which is chosen over the whole period. Thus, the costs with respect to this strategy are also calculated over the entire period.

Since the cost function acceleration is independent of the strategy of the steering angle acceleration ω_u and only the acceleration a causes costs $J_{acc,i}$, there are also no changes in the simulation results of the other two scenarios, since the main difference between the scenarios is the reasonable strategy choice of the steering angle acceleration ω. The determined and simulated values for the different strategies of acceleration in Table A.23 and Table A.24 in Appendix A.8 confirm this.

ID	u_veh1	u_veh2	J_veh1	J_veh2
AC1.1	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
AC1.2	[-0.004, 0.0], [-0.005.0, 0.0], [-0.006, 0.0]	[-0.004, 0.0], [-0.005.0, 0.0], [-0.006, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
AC1.3	[-0.01, 0.0], [-0.02, 0.0], [-0.03, 0.0]	[-0.01, 0.0], [-0.02, 0.0], [-0.03, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
AC1.4	[-0.04, 0.0], [-0.05, 0.0], [-0.06, 0.0]	[-0.04, 0.0], [-0.05, 0.0], [-0.06, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
AC1.5	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-0.03 -0.03 -0.03] [-0.03 -0.03 -0.03] [-0.03 -0.03 -0.03]	[-0.03 -0.03 -0.03] [-0.03 -0.03 -0.03] [-0.03 -0.03 -0.03]
AC1.6	[0.0, 0.5], [0.0, 0.5], [0.0, 0.5]	[0.0, 0.5], [0.0, 0.5], [0.0, 0.5]	[-0.009 -0.009 -0.009] [-0.009 -0.009 -0.009] [-0.009 -0.009 -0.009]	[-0.009 -0.009 -0.009] [-0.009 -0.009 -0.009] [-0.009 -0.009 -0.009]
AC1.7	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[-0.03 -0.03 -0.03] [-0.009 -0.009 -0.009] [-0. -0. -0.]	[-0.03 -0.03 -0.03] [-0.009 -0.009 -0.009] [-0. -0. -0.]
AC1.8	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[-0.09 -0.09 -0.09] [-0.02 -0.02 -0.02] [-0. -0. -0.]	[-0.03 -0.03 -0.03] [-0.009 -0.009 -0.009] [-0. -0. -0.]

Table 6.8.: Visualization of the simulation results of the first scenario for the cost function acceleration work

6.2. Simulation for combinations of cost functions

Based on the simulation results of the cost function collision energy from Chapter 6.1.1 and since there is no exact match of points due to the consideration of points in the simulation, there is no cost for the cost function collision energy. Thus, it does not make sense to form combinations with the cost function collision energy. Based on these findings, the following reasonable combinations of the safety-relevant cost functions remain:

- cost function center lane offset and cost function Euclidean distance
- cost function center lane offset, cost function Euclidean distance and cost function obstacle distance
- cost function center lane offset and cost function TTC
- cost function center lane offset, cost function TTC and cost function obstacle distance

Considering the defined weighting factors from Chapter 5.6, the following formulas result for the defined combinations of cost functions:

$$J_{clo,eucl} = J_{clo,i} \cdot K_{l,clo} + J_{eucl,i} \cdot K_{l,eucl} \tag{6.10}$$

$$J_{clo,eucl,obs} = J_{clo,i} \cdot K_{l,clo} + J_{eucl,i} \cdot K_{l,eucl} + J_{obs,i} \cdot K_{l,obs} \tag{6.11}$$

$$J_{clo,ttc} = J_{clo,i} \cdot K_{l,clo} + J_{TTC,i} \cdot K_{l,TTC} \tag{6.12}$$

$$J_{clo,TTC,obs} = J_{clo,i} \cdot K_{l,clo} + J_{TTC,i} \cdot K_{l,TTC} + J_{obs,i} \cdot K_{l,obs} \tag{6.13}$$

The defined combinations of the cost functions can also be combined with the cost $J_{vel,i}$ and weighting factor $K_{l,vel}$ for the quadratic velocity, the cost $J_{acc,i}$ and weighting factor $K_{l,acc}$ for the acceleration, and the cost $J_{ster,i}$ and weighting factor $K_{l,ster}$ for the steering angle combined. Within the scope of this work, no simulations are performed regarding the weighting of combined cost functions, since the weighting of the functions would result in the basis of many assumptions. Furthermore, due to the numerous weighting possibilities, there would be numerous simulation possibilities, which would be beyond the scope of this work. However, as justified earlier, the cost functions that avoid collisions should be weighted higher than the cost functions that address passenger comfort and regulatory compliance. Chapter 8 takes the reference to the simulation and weighting of the cost function and gives an outlook on how this can be addressed in further work.

7. Simulation of the optimized cost functions in a normal form game

In this Chapter, the optimized cost functions from Chapter 5.5 are simulated individually and evaluated and assessed with respect to the scenarios from Chapter 5.2. The optimized cost functions are simulated and evaluated using the same strategies as the cost functions from Chapter 5.3 and Chapter 5.4. As can be seen from the results, it is difficult to interpret the calculated NE from the payoff matrices of the respective vehicles. The reason for this is that the uncertainties make interpretation difficult and the cost functions also show individual weaknesses. Therefore, an interpretation of the NE in the context of this work is not expedient and is therefore not considered further.

7.1. Simulation for the optimized cost function collision energy

In the simulation results for the optimized cost function collision energy in the Appendix B.1, it is clear that in all payoff matrices of the considered maneuvers of the three scenarios over the time horizon $t_{horizon}$, as in Chapter 6.1.1, there is no cost to the two vehicles and thus the two vehicles do not collide according to the payoff matrices and the formulated cost function. As explained earlier, the reason for this is that the simulation only considers points as states for the vehicle and therefore a collision only occurs if the states overlap exactly. This is unlikely due to the different parameters, complexity and the defined uncertainties and is therefore reflected in the simulation results for the cost function collision energy and in the simulation results for the optimized cost function collision energy. Accordingly, a comparison between the simulation results of the two cost functions for collision energy is not possible within the scope of this work.

7.2. Simulation for optimized cost function TTC

For the optimized cost function TTC, the same conditions and assumptions apply as for the cost function TTC from Chapter 6.1.2. Thus, also for this cost function, costs arise for the respective vehicles if the shortest distance d_s between the two vehicles falls below the safety distance $d_{TTC,safe}$ of 3.4 m.

For the second scenario, in which the two vehicles drive behind each other according to Chapter 5.2, it can be derived from Table B.5 that by optimizing the cost function already at an earlier time t in the defined time horizon the safety distance $d_{TTC,safe}$ is undercut by the shortest distance between the two vehicles d_s (cf. maneuver TTC2.2 and maneuver TTC2.2opt). This effect occurs only for the strategies that have already yielded costs for the cost function TTC. If the maneuver TTC2.5 is considered, which represents an overtaking maneuver of the first vehicle, it can be interpreted from the results that vehicle 1 is closer to vehicle 2 in this overtaking maneuver due to the higher costs than in the cost function TTC, whose distance does not correspond to the worst case. The explained effects for the second scenario of the optimized cost function obstacle distance also occur in the other two scenarios and can therefore be interpreted in the same way. The simulation results for scenario 1 and scenario 3 can be seen in Table B.4 and Table B.6in Appendix B.5.

When comparing the simulation results between cost function TTC and the optimized cost function TTC, for example, when comparing maneuvers TTC1.1 and TTCopt1.1, it is noticeable that the costs are lower for the optimized cost function and consequently cannot represent the expected worst case. These results are due to the fact that the costs in relation to the worst case TTC according to Formula 5.22 and Formula 5.23 consist of the worst case of the minimum distance and the worst case of the maximum speed.

7.3. Simulation for optimized center lane offset

For the optimized cost function center lane offset from Chapter 5.3.3 the same conditions and assumptions apply as simulation for the cost function center lane offset from Chapter 6.1.3. Therefore, also for this cost function a tolerable distance factor to the lane center $K_{tol,clo}$ of applies, so that a defined lane width of 3.5 m results in a tolerance range of 0.4375 m to the left and right side of the lane center. Thus, also for this cost function, costs arise for the respective vehicle if the distance to the lane center exceeds the tolerable distance $K_{tol,clo}$.

For the first scenario it can be derived from Table B.7that by optimizing the cost function already at an earlier time t in the defined time horizon the tolerable distance factor to the lane center

$K_{tol,clo}$ is exceeded, since the optimization determines the largest possible distance from the lane center, which in the case of this cost function represents the worst case. Also with respect to the strategy of the steering angle acceleration ω_u it is noticeable that, for example, in the comparison of maneuver CLO1.5 and maneuver CLOopt1.4 with the optimized cost function already for lower strategies of the steering angle acceleration ω_u higher costs arise for the respective vehicle, since the tolerable distance factor to the lane center $K_{tol,clo}$ is exceeded at an earlier time. Similarly for the cost considering the strategy of acceleration a_u. Again, by optimizing the cost function in the worst case, the distance to the center lane becomes larger than in the simulation results of the cost function center lane offset from Chapter 6.1.3. However, these differences to the cost function center lane offset are so small that they are not visually noticeable, which is why a visual representation of the trajectories of the cost function is omitted in the comparison. Figure 7.1 illustrates exemplarily the simulation of the cost function center lane offset of the maneuver CLOopt2.6.

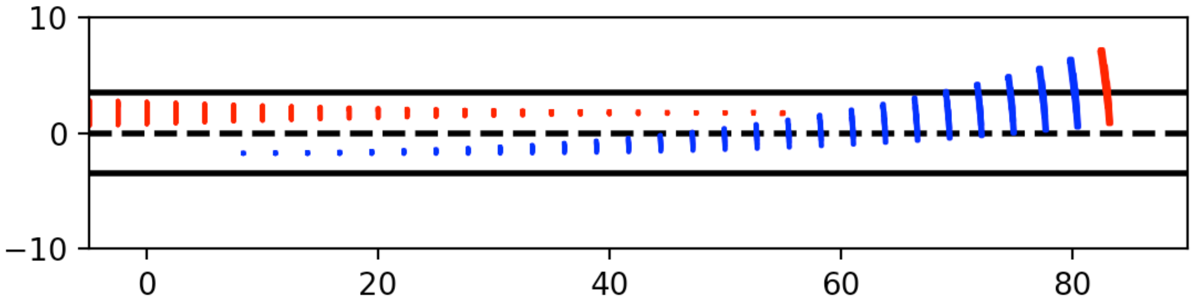

Table 7.1.: Visualization of the simulation results of the maneuver CLOopt2.6 of the second scenario in relation to the cost function obstacle distance

The effects explained for the first scenario of the optimized cost function center lane offset also occur in the other two scenarios and can therefore be interpreted in the same way. The simulation results for scenario 2 and scenario 3 can be seen in Table B.8 and Table B.9in Appendix B.3.

7.4. Simulation for optimized cost function Euclidean distance

For the optimized cost function Euclidean distance, the same conditions and assumptions apply as for the simulation results cost function Euclidean distance from Chapter 6.1.4. Thus, also for this cost function, costs arise for the respective vehicles if the shortest distance d_s between the two vehicles falls below the safety distance $d_{eucl,safe}$ of 3.4 m.

For the second scenario, in which the two vehicles drive behind each other according to Chapter 5.2, it can be derived from Table B.11 that by optimizing the cost function already at an earlier time t in the defined time horizon the safety distance $d_{eucl,safe}$ is undercut by the shortest distance between the two vehicles d_s (cf. maneuver ED2.2 and maneuver ED2.2opt). This effect occurs only for the strategies that have already yielded costs for the Euclidean distance cost function. If the maneuver TTC2.5 is considered, which represents an overtaking maneuver of the first vehicle, it can be interpreted from the results that vehicle 1 is closer to vehicle 2 in this overtaking maneuver than in the cost function TTC, whose distance does not correspond to the worst case, due to the higher costs. The explained effects for the second scenario of the optimized cost function Euclidean distance also occur in the other two scenarios and can therefore be interpreted in the same way. The simulation results for scenario 1 and scenario 3 can be seen in Table B.10 and Table B.12in Appendix B.4.

7.5. Simulation for optimized cost function obstacle distance

For the optimized cost function obstacle distance, the same conditions and assumptions apply as for the cost function obstacle distance from Chapter 6.1.5. Thus, this cost function also incurs costs for the respective vehicle if the shortest distance $d_{obs,s}$ to the obstacle falls below the safety distance $d_{obs,safe}$.

For the first scenario, it can be deduced from Table B.13 that by optimizing the cost function, the safety distance $d_{obs,safe}$ from the shortest distance to the obstacle $d_{obs,s}$ is undercut already at an earlier time t in the defined time horizon. Also with respect to the strategy of the steering angle acceleration ω_u it is noticeable that, for example, in the comparison of maneuver OBS1.4 and maneuver OBSopt1.4 with the optimized cost function, already for lower strategies of the steering angle acceleration ω_u costs arise for the respective vehicle, which falls below the safety distance to the obstacle $d_{obs,safe}$. The situation is similar for the cost considering the strategy of acceleration a_u. Again, the shortest distance $d_{obs,s}$ due to the optimization of the cost function in the worst case is smaller than in the simulation results of the cost function obstacle distance from Chapter 6.1.5. However, these differences from the cost function obstacle distance are so small that they are not visually apparent, which is why we do not present a visual representation of the trajectories of the cost function in the comparison.

The effects explained for the first scenario of the optimized cost function obstacle distance also occur in the other two scenarios and can therefore be interpreted in the same way. The simulation results for scenario 2 and scenario 3 can be seen in Table B.14 and Table B.14in Appendix B.5.

7.6. Simulation for optimized cost function quadratic velocity

The limit speed tolerance v_T already explained in Chapter 5.4.1 and used in Chapter 7.6 is also defined for the optimization with 1 m/s, so that costs are only incurred for the respective vehicle if the deviation of the speed of the respective vehicle from the limit speed is greater than the limit speed tolerance range. With a defined speed limit v_L of 27.7778 m/s, only costs for speeds outside the tolerance range of 26.7778 m/s to 28.7778 m/s are calculated for the respective vehicle. As explained in chapter 5.4.1, the factor K_{vel} was introduced for a low speed outside the defined tolerance range to be able to weight the costs $J_{vel},_i$ for a low speed lower. In the context of the simulation, this factor K_{vel} is defined as 0.1, so that the impact of the differences can be clearly identified and evaluated from the results.

ID	u_veh1	u_veh2	J_veh1	J_veh2
VQopt1.8	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-9.80 -9.80 -9.80] [-9.80 -9.80 -9.80] [-9.80 -9.80 -9.80]
VQopt1.9	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[-28.71 -28.71 -28.71] [-0.97 -0.97 -0.97] [-0. -0. -0.]	[-2.04 -3.90 -9.80] [-2.04 -3.90 -9.80] [-2.04 -3.90 -9.80]
VQopt1.10	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[-94.56 -94.56 -94.56] [-7.18 -7.18 -7.18] [-0. -0. -0.]	[-2.10 -3.91 -12.96] [-2.10 -3.91 -12.96] [-2.10 -3.91 -12.96]
VQopt1.11	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-98.02 -98.02 -98.02] [-98.02 -98.02 -98.02] [-98.02 -98.02 -98.02]	[-0.47 -0.47 -0.47] [-0.47 -0.47 -0.47] [-0.47 -0.47 -0.47]
VQopt1.12	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0.47 -0.47 -0.47] [-0.47 -0.47 -0.47] [-0.47 -0.47 -0.47]	[-64.35 -64.35 -64.35] [-64.35 -64.35 -64.35] [-64.35 -64.35 -64.35]

Table 7.2.: Extract from the simulation results of the first scenario for the cost function quadratic velocity from Table B.16 in Appendix B.6

The simulation results of the cost function quadratic velocity and the optimized cost function quadratic velocity show only small differences in the second decimal place (cf. VQ1.8 and VQopt1.8). Only higher deviations of the velocity of the vehicle from the defined speed limit v_L result in higher costs. This is the case both for lowering a lower speed (cf. VQopt1.12, VQopt2.5, VQopt3.5 vehicle 2) than the speed limit and for increasing a higher speed (cf. VQopt1.11, VQopt2.4, VQopt3.4 vehicle 1) than the speed limit. Furthermore, all other effects, which result in the simulation of the cost function quadratic velocity in Chapter 7.6, can also be adopted for the optimized cost function of quadratic velocity. The higher costs were to be expected due to the consideration of the worst case and thus in this case of the maximum possible velocity.

7.7. Simulation for optimized cost function steering angle

As with the simulation of the cost function steering angle in Chapter 6.1.7, individual simulation results can be expected for the simulation of the optimized cost function steering angle, which was defined in Chapter 5.5.6, both when changing the strategy of the steering angular velocity ω_u and when changing the strategy of the acceleration a_u. In the following Table 7.3 with simulation results of the optimized cost function steering angle for the first scenario, it becomes clear when comparing the results for the maneuver STopt1.1 that, as with the cost function steering angle in Chapter 6.1.7, a strategy without steering angle velocity ω_u results in costs for the respective vehicles. The reason for this is the consideration of uncertainties in the strategy and initial state for the steering angle as defined in Chapter 5.1. Since the two vehicles drive at different speeds, different results are obtained due to the dependence of the cost function steering angle on the speed according to Formula 5.37. The higher the speed of the vehicle, the higher the cost of the respective vehicle.

Taking into account the strategy with acceleration a_u, it becomes clear, among other things, in Maneuver STopt1.2 in Table 7.3 that the acceleration has an effect on the costs of the cost function steering angle and thus the same effects occur as for the cost function steering angle in Chapter 6.1.7. This can also be justified with the fact that the constant acceleration contributes to the fact that the speed of the vehicles increases. Due to the speed dependency of the cost function according to Formula 5.37, the costs for the respective vehicle increase under consideration of the strategy of acceleration. Considering a strategy regarding the steering angle acceleration ω_u, comparing the maneuvers STopt1.3 to STopt1.10 with the maneuvers ST1.3 to ST1.10 of the cost function steering angle, it is noticed that the simulation results of the optimized cost function do not reflect the expected higher costs of the worst case, although the worst case of the highest possible steering angle and the worst case of the highest possible velocity should cause higher costs according to Chapter 5.5.6.

The explained effects for the first scenario of the optimized cost function steering angle also occur in the other two scenarios and can therefore be interpreted in the same way. The simulation

ID	u_veh1	u_veh2	J_veh1	J_veh2
STopt1.1	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-28.24 -28.24 -28.24] [-28.24 -28.24 -28.24] [-28.24 -28.24 -28.24]	[-22.93 -22.93 -22.93] [-22.93 -22.93 -22.93] [-22.93 -22.93 -22.93]
STopt1.2	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-31.49 -31.49 -31.49] [-31.49 -31.49 -31.49] [-31.49 -31.49 -31.49]	[-25.81 -25.81 -25.81] [-25.81 -25.81 -25.81] [-25.81 -25.81 -25.81]
STopt1.3	[-0.01, 0.0], [-0.03, 0.0], [-0.05, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-332.64 -332.64 -332.64] [-1050.02 -1050.02 -1050.02] [-1767.24 -1767.24 -1767.24]	[-22.93 -22.93 -22.93] [-22.93 -22.93 -22.93] [-22.93 -22.93 -22.93]
STopt1.4	[-0.001, 0.0], [-0.005, 0.0], [-0.009, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-18.36 -18.36 -18.36] [-154.16 -154.16 -154.16] [-296.91 -296.91 -296.91]	[-22.93 -22.93 -22.93] [-22.93 -22.93 -22.93] [-22.93 -22.93 -22.93]
STopt1.5	[-0.01, 1.0], [-0.03, 1.0], [-0.05, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-384.14 -384.14 -384.14] [-1210.60 -1210.60 -1210.60] [-2037.31 -2037.31 -2037.31]	[-22.93 -22.93 -22.93] [-22.93 -22.93 -22.93] [-22.93 -22.93 -22.93]
STopt1.6	[-0.001, 1.0], [-0.005, 1.0], [-0.009, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-20.91 -20.91 -20.91] [-178.33 -178.33 -178.33] [-342.95 -342.95 -342.95]	[-22.93 -22.93 -22.93] [-22.93 -22.93 -22.93] [-22.93 -22.93 -22.93]
STopt1.7	[-0.01, 0.0], [-0.03, 0.0], [-0.05, 0.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-332.64 -332.64 -332.64] [-1050.02 -1050.02 -1050.02] [-1767.24 -1767.24 -1767.24]	[-25.81 -25.81 -25.81] [-25.81 -25.81 -25.81] [-25.81 -25.81 -25.81]
STopt1.8	[-0.001, 0.0], [-0.005, 0.0], [-0.009, 0.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-18.36 -18.36 -18.36] [-154.16 -154.16 -154.16] [-296.91 -296.91 -296.91]	[-25.81 -25.81 -25.81] [-25.81 -25.81 -25.81] [-25.81 -25.81 -25.81]
STopt1.9	[-0.01, 1.0], [-0.03, 1.0], [-0.05, 1.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-384.14 -384.14 -384.14] [-1210.60 -1210.60 -1210.60] [-2037.31 -2037.31 -2037.31]	[-25.81 -25.81 -25.81] [-25.81 -25.81 -25.81] [-25.81 -25.81 -25.81]
STopt1.10	[-0.001, 1.0], [-0.005, 1.0], [-0.009, 1.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-20.91 -20.91 -20.91] [-178.33 -178.33 -178.33] [-342.95 -342.95 -342.95]	[-25.81 -25.81 -25.81] [-25.81 -25.81 -25.81] [-25.81 -25.81 -25.81]

Table 7.3.: Simulation results of the first scenario for the optimized cost function steering angle

results for scenario 2 and scenario 3 can be seen in Table B.20 and Table B.21in Appendix B.7.

7.8. Simulation for optimized cost function acceleration work

The simulation of the optimized cost function acceleration work from Chapter 5.5.6 generates the same results as the simulation of the cost function acceleration from Chapter 6.1.8. The reason for the same results lies in the formula 5.46, which for the reason of explanation is also presented in formula 7.1 is shown.

$$J_{acc,i} = \frac{1}{2} \cdot m_i \cdot \Delta v_i^2 \tag{7.1}$$

The Formula 7.1 shows that if the factor of the mass of the respective vehicle m_i and/or the factor of the speed difference Δv_i changes, the cost of the acceleration work $J_{acc,i}$ for the respective vehicle i changes. The mass remains identical due to the comparability of the defined scenarios and vehicle. The velocity change Δv_i of a vehicle i is calculated from the amount of difference between the defined velocity in initial state v_i and the velocity at the end of the time horizon $t_{horizon}$ v_{endi}. The velocity v_{endi} at the end of the time horizon $t_{horizon}$ is calculated by the following Formula 7.2:

$$v_{end,i} = a_{v1} \cdot t_{horizon} + v_i \tag{7.2}$$

Therefore, provided that the defined speed is changed in the initial state, as in the case of optimization and consideration of the highest possible speed, the results (see tables in Appendix B.8) show no changes in the cost $J_{acc,i}$ compared to the cost of the non-optimized speed in the Tables in Appendix A.8.

8. Conclusion and further work

To create a normal form game, discrete control functions have been used in this work instead of continuous control functions as generally used in differential games by Zanardi et al. (2021c) and Mylvaganam et al. (2017). For the different discrete control variables of the discrete control functions, reachability analysis has been used to define the different reachable states of the player. In addition, the discrete control functions allowed games to be considered and solved in a normal way, rather than the complex and time-consuming solution process of differential games. Based on the defined reachable states, the cost for each player could be estimated. Here, the reachability analysis also made it possible to take into account the uncertainty in the initial state and in the strategy of the players. In order to determine the costs for each player, two groups of cost functions have been defined in this work, one group containing cost functions to prevent collisions and the other group containing running costs that take into account the speed and comfort of the occupant. These defined cost functions have been simulated and evaluated using three defined driving scenarios from Chapter 5.2, which include different driving maneuvers. The simulation results in this thesis have shown that the defined cost functions are useful cost functions that can be used in games with two players to avoid collisions. The cost function Time-To-Collision (TTC) in Chapter 5.3.2 determines the time remaining until the players can potentially collide. The cost function center lane offset from Chapter 5.3.3 calculates costs based on the distance of the vehicle from the center of the lane of the defined vehicle. The Euclidean distance cost function, which was defined in Chapter 5.3.4, determines the shortest distance between two vehicles that are in the same lane and traveling in the same direction, thus addressing possible rear-end collisions. Another distance cost function is the cost function obstacle distance from Chapter 5.3.5, which is based on the shortest distance between the respective vehicle and an obstacle.

In addition to these cost functions, other defined cost functions have proven to be useful, which have no direct influence on the avoidance of collisions, but had to be taken into account for safety-related reasons. These cost functions include the cost function quadratic velocity in Chapter 5.4.1 which has to be taken into account due to speed limits imposed by legal regulations. The other two cost functions of the steering angle change in Chapter 5.4.2 and the acceleration work in Chapter 5.4.3 were defined for reasons of comfort for the occupant.

A cost function that did not give the expected results in the context of this thesis is the cost function collision energy in Chapter 5.3.1. Since only points are considered as states of the vehicle in the context of this thesis, a collision between the two vehicles only occurs when the states exactly overlap. This is unlikely due to the different parameters, complexity and defined uncertainties and is therefore reflected in the simulation results for the cost function collision energy. Beyond the definition and simulation of the cost functions, this paper also contains a proposal of possible combinations of these defined cost functions that can be considered in further work. With respect to the obstacle distance cost function, but also with respect to the other cost functions, further work can determine the calculation of costs in terms of cost function combinations. In particular, in the case of the first and third scenarios according to Chapter 5.2, combinations between cost functions such as TTC or Euclidean distance with the obstacle distance cost function are interesting to evaluate whether it is more expensive to collide with the vehicle or the obstacle, or whether there is a strategy for both vehicles to avoid colliding with both the obstacle and the other vehicle. This would also require a specification of the weighting of the cost function.

Furthermore, in the present work, the worst cases for each pair of controls included in the reachable states were estimated according to Chapter 4.5. Depending on the cost function, the cost functions were either maximized or minimized. As part of the nonlinear optimization of the cost functions, optimization functions were defined in which the respective cost functions were adjusted with respect to the worst cases. The simulation results of these optimized cost functions generally showed the higher costs expected when taking the worst cases into account. Individual deviations that arose in the simulation can be attributed to the uncertainties taken into account. In addition, for the cost functions that take the shortest distance into account, the shortest distance was considered as a minimum and the speed was considered as a maximum in the calculation of the costs due to the speed dependency, as a result of which some cost functions did not deliver simulation results that could be interpreted in a comprehensible manner.

A concrete comparison between the results of the present work and the work of the Zanardi et al. (2021c) and Mylvaganam et al. (2017) is not readily possible because different approaches are taken, as explained in Chapter 1 and in the state of the art in Chapter 3.

In further work, the defined scenarios could be extended with respect to other layers according to DLR (2019) and Schuldt (2017), for example, weather conditions and the resulting larger distances between vehicles could be added. In addition, the cost functions could also be supplemented and concreted. Ozbay (2008) proposes, among other things, an approach to identify possible appropriate situations that cannot be captured by the general calculation of TTC from Chapter 5.3.2, but which may nevertheless have potential conflicts. This approach includes, but is not limited to, those based on equations of motion and the assumption of constant acceleration. Whether a conflict would occur is based on consideration of the trajectory parameters of preceding and following vehicles, including relative distance, relative speed, and relative acceleration.

For the purposes of this work, only static form games are considered. Further work can consider extensive form games explained in Chapter 2.1.2. In these, players would then not even change or maintain their strategies for the entire time horizon $t_{horizon}$, but at any point within that horizon based on the strategies of the other players.

Of interest for further work are uncertainties in the state of the obstacle with respect to the cost function obstacle distance, as they have already been considered for the vehicles and strategies in this work. Thus also probability predictions at which exact point the obstacle is located can be integrated into the cost function. Furthermore, it would also be interesting if not only static objects are considered, but also dynamic objects that do not represent a vehicle. Examples for this would be other road users like pedestrians, cyclists or even animals, which can be on the roadway. The dynamics of these obstacles can also be described by differential equations.

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A. Simulation result for the cost functions

A.1. Simulation result for the cost function collision energy

ID	u_veh1	u_veh2	J_veh1	J_veh2
CE1.1	[-0.004, 0.0], [-0.005, 0.0], [-0.006, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
CE1.2	[-0.007, 0.0], [-0.008, 0.0], [-0.009, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
CE1.3	[-0.004, 0.0], [-0.006, 0.0], [-0.008, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
CE1.4	[-0.004, 1.0], [-0.005, 1.0], [-0.006, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
CE1.5	[-0.007, 1.0], [-0.008, 1.0], [-0.009, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
CE1.6	[-0.004, 1.0], [-0.006, 1.0], [-0.008, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]

Table A.1.: Simulation results of the first scenario for the cost function collision energy

ID	u_veh1	u_veh2	J_veh1	J_veh2
CE2.1	[-0.0, 0.0], [0.0, 0.0], [0.00, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
CE2.2	[-0.0, 1.0], [0.0, 1.0], [0.00, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
CE2.3	[-0.0, 0.0], [0.0, 0.0], [0.00, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]

Table A.2.: Simulation results of the second scenario for the cost function collision energy

ID	u_veh1	u_veh2	J_veh1	J_veh2
CE3.1	[0.0, 0.0], [0.0, 0.0], [0.00, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
CE3.2	[0.07, 0.0], [0.08, 0.0], [0.09, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
CE3.3	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
CE3.4	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
CE3.5	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
CE3.6	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
CE3.7	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
CE3.8	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
CE3.9	[0.01, 1.0], [0.02, 1.0], [0.03, 1.0]	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
CE3.11	[0.04, 1.0], [0.05, 1.0], [0.06, 1.0]	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
CE3.12	[0.01, 1.0], [0.02, 1.0], [0.03, 1.0]	[0.01, 1.0], [0.02, 1.0], [0.03, 1.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
CE3.13	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]

Table A.3.: Simulation results of the third scenario for the cost function collision energy

A.2. Simulation result for the cost function Time-to-Collision

ID	u_veh1	u_veh2	J_veh1	J_veh2
TTC1.1	[-0.01, 0.0], [-0.02, 0.0], [-0.03, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [3623.84 3610.18 4555.80]	[-0. -0. -0.] [-0. -0. -0.] [2935.34 2924.28 3690.27]
TTC1.2	[-0.03, 0.0], [-0.04, 0.0], [-0.05, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[2706.69 2728.59 3666.63] [6086.37 5116.13 5125.57] [5827.68 5784.06 5803.04]	[2192.56 2210.24 2969.99] [4930.09 4144.11 4151.59] [4720.57 4685.17 4700.50]
TTC1.3	[-0.01, 1.0], [-0.02, 1.0], [-0.03, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
TTC1.4	[-0.03, 1.0], [-0.04, 1.0], [-0.05, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [2266.36 1477.64 1479.80] [3534.11 3550.76 3545.93]	[-0. -0. -0.] [1726.87 1119.03 1120.78] [2686.82 2699.70 2696.01]
TTC1.5	[-0.01, 0.0], [-0.02, 0.0], [-0.03, 0.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[9743.11 11877.12 7336.70] [14767.54 12252.47 14743.60] [12225.71 10917.65 10931.95]	[9033.56 10988.68 6809.87] [13280.19 10962.39 13255.40] [10815.65 9693.76 9704.63]
TTC1.6	[-0.03, 0.0], [-0.04, 0.0], [-0.05, 0.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[12118.10 12181.01 12260.79] [9261.10 11090.78 9878.72] [9025.62 9030.16 9077.01]	[10718.61 10774.26 10848.95] [8082.54 9733.51 8704.09] [7884.91 7887.14 7929.43]
TTC1.7	[-0.01, 1.0], [-0.02, 1.0], [-0.03, 1.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-0. -0. -0.] [-0. -0. -0.] [4860.41 3891.81 4887.46]	[-0. -0. -0.] [-0. -0. -0.] [3970.35 3180.33 3991.76]
TTC1.8	[-0.03, 1.0], [-0.04, 1.0], [-0.05, 1.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[4828.85 4836.59 4816.23] [6451.62 6449.83 6412.25] [6076.13 6064.63 6049.58]	[3944.58 3950.92 3934.29] [5266.22 5264.77 5234.21] [4955.73 4946.41 4934.14]

Table A.4.: Simulation results of the first scenario for the cost function Time-to-Collision

ID	u_veh1	u_veh2	J_veh1	J_veh2
TTC2.1	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
TTC2.2	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[1504.90 829.61 822.69] [1460.32 938.13 921.99] [1371.78 381.30 1468.99]	[999.82 549.34 544.71] [970.55 620.91 610.24] [911.76 251.58 976.33]
TTC2.3	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
TTC2.4	[0.003, 1.0], [0.004, 1.0], [0.005, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
TTC2.5	[0.001, 1.0], [0.002, 1.0], [0.003, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[1485.25 860.46 1398.51] [-0. -0. -0.] [-0. -0. -0.]	[986.66 569.74 929.51] [-0. -0. -0.] [-0. -0. -0.]

Table A.5.: Simulation results of the second scenario for the cost function Time-to-Collision

ID	u_veh1	u_veh2	J_veh1	J_veh2
TTC3.1	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
TTC3.2	[0.07, 0.0], [0.08, 0.0], [0.09, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[855.79 861.08 874.51] [935.38 941.28 -0.] [-0. -0. -0.]	[693.24 697.45 708.30] [757.66 762.38 -0.] [-0. -0. -0.]
TTC3.3	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[1455.04 1452.91 1480.37] [1487.40 1497.05 1488.79] [1624.93 1646.71 1622.89]	[1178.68 1176.89 1199.17] [1204.90 1212.64 1205.99] [1316.22 1333.78 1314.53]
TTC3.4	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [801.99 795.40 782.08] [1629.56 1621.62 1613.29]	[-0. -0. -0.] [649.59 644.28 633.40] [1319.99 1313.49 1306.69]
TTC3.5	[0.008, 0.0], [0.009, 0.0], [0.01, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
TTC3.6	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.07, 0.0], [0.08, 0.0], [0.09, 0.0]	[1497.75 1686.02 924.22] [1514.79 1710.31 920.24] [1538.75 1750.81 944.06]	[1213.07 1365.64 748.62] [1226.89 1385.34 745.43] [1246.32 1418.16 764.76]
TTC3.7	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[1558.53 1450.11 1432.56] [1547.96 1450.77 1438.74] [1525.23 1443.58 1447.42]	[1262.37 1174.55 1160.32] [1253.82 1175.09 1165.34] [1235.43 1169.29 1172.39]
TTC3.8	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[-0. 904.25 761.80] [-0. 906.78 763.42] [-0. 875.19 1674.82]	[-0. 732.34 617.02] [-0. 734.43 618.36] [-0. 708.88 1356.63]
TTC3.9	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.008, 0.0], [0.009, 0.0], [0.01, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
TTC3.10	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.04, 1.0], [0.05, 1.0], [0.06, 1.0]	[852.90 792.56 800.69] [854.46 792.08 797.87] [821.22 774.87 800.54]	[752.97 699.65 706.86] [754.38 699.26 704.41] [725.07 684.10 706.80]
TTC3.11	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.01, 1.0], [0.02, 1.0], [0.03, 1.0]	[1542.70 1293.29 1054.45] [1548.77 1298.62 1058.53] [1487.25 1240.67 1007.83]	[1361.97 1141.62 930.85] [1367.40 1146.38 934.49] [1313.14 1095.28 889.78]
TTC3.12	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[0.04, 1.0], [0.05, 1.0], [0.06, 1.0]	[403.05 314.77 346.02] [447.56 333.97 344.25] [396.76 312.86 347.28]	[329.25 257.14 282.66] [365.61 272.83 281.22] [324.11 255.58 283.69]
TTC3.13	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[0.01, 1.0], [0.02, 1.0], [0.03, 1.0]	[858.61 694.89 513.62] [913.58 748.79 564.15] [850.40 686.87 506.23]	[701.30 567.66 419.58] [746.21 611.70 460.86] [694.59 561.11 413.54]
TTC3.14	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[684.03 551.18 476.58] [721.34 586.30 508.96] [678.50 546.04 471.91]	[512.61 413.11 357.20] [540.58 439.44 381.48] [508.47 409.26 353.70]
TTC3.15	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[383.00 359.56 368.93] [406.34 374.78 370.43] [379.86 357.75 369.23]	[287.02 269.46 276.51] [304.51 280.87 277.64] [284.66 268.10 276.73]
TTC3.16	[0.008, 0.0], [0.04, 0.0], [0.07, 0.0]	[-0.000, 0.0], [-0.00, 0.0], [-0.000, 0.0]	[937.08 -0. -0.] [1464.07 1465.97 1470.78] [841.24 836.39 828.06]	[759.07 -0. -0.] [1185.87 1187.42 1191.32] [681.39 677.41 670.74]

Table A.6.: Simulation results of the third scenario for the cost function Time-to-Collision

A.3. Simulation result for the cost function center lane offset

ID	u_veh1	u_veh2	J_veh1	J_veh2
CLO1.1	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-83016.86 -83016.86 -83016.86] [-82633.20 -82633.20 -82633.20] [-81780.68 -81780.68 -81780.68]	[-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96]
CLO1.2	[-0.01, 0.0], [-0.03, 0.0], [-0.05, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-84124.55 -84124.55 -84124.55] [-216615.02 -216615.02 -216615.02] [-312745.46 -312745.46 -312745.46]	[-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96]
CLO1.3	[-0.001, 0.0], [-0.01, 0.0], [-0.03, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-75992.56 -75992.56 -75992.56] [-82324.18 -82324.18 -82324.18] [-216809.78 -216809.78 -216809.78]	[-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96]
CLO1.4	[-0.008, 0.0], [-0.009, 0.0], [-0.01, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-72929.92 -72929.92 -72929.92] [-77150.33 -77150.33 -77150.33] [-83906.62 -83906.62 -83906.62]	[-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96]
CLO1.5	[-0.004, 0.0], [-0.005, 0.0], [-0.006, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-56511.21 -56511.21 -56511.21] [-60167.62 -60167.62 -60167.62] [-63132.50 -63132.50 -63132.50]	[-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96]
CLO1.6	[-0.01, 1.0], [-0.03, 1.0], [-0.05, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-101582.38 -101582.38 -101582.38] [-282376.29 -282376.29 -282376.29] [-394580.40 -394580.40 -394580.40]	[-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96]
CLO1.7	[-0.001, 1.0], [-0.01, 1.0], [-0.03, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-80004.47 -80004.47 -80004.47] [-105844.15 -105844.15 -105844.15] [-282205.97 -282205.97 -282205.97]	[-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96]
CLO1.8	[-0.008, 1.0], [-0.009, 1.0], [-0.01, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-86605.31 -86605.31 -86605.31] [-94833.96 -94833.96 -94833.96] [-104465.37 -104465.37 -104465.37]	[-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96]
CLO1.9	[-0.004, 1.0], [-0.005, 1.0], [-0.006, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-62608.19 -62608.19 -62608.19] [-70613.70 -70613.70 -70613.70] [-71268.80 -71268.80 -71268.80]	[-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96]
CLO1.10	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-95495.92 -95495.92 -95495.92] [-89779.41 -89779.41 -89779.41] [-87747.39 -87747.39 -87747.39]	[-4312.18 -5212.02 -4633.39] [-4312.18 -5212.02 -4633.39] [-4312.18 -5212.02 -4633.39]

Table A.7.: Simulation results of the first scenario for the cost function center lane offset

ID	u_veh1	u_veh2	J_veh1	J_veh2
CLO2.1	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-3145.77 -3145.77 -3145.77] [-2179.17 -2179.17 -2179.17] [-3702.41 -3702.41 -3702.41]	[-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96]
CLO2.2	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-6689.33 -6689.33 -6689.33] [-6407.11 -6407.11 -6407.11] [-5655.56 -5655.56 -5655.56]	[-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96]
CLO2.3	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-6689.33 -6689.33 -6689.33] [-6407.11 -6407.11 -6407.11] [-5655.56 -5655.56 -5655.56]	[-4312.18 -5212.02 -4633.39] [-4312.18 -5212.02 -4633.39] [-4312.18 -5212.02 -4633.39]
CLO2.4	[0.003, 1.0], [0.004, 1.0], [0.005, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-31276.98 -31276.98 -31276.98] [-46727.16 -46727.16 -46727.16] [-58065.92 -58065.92 -58065.92]	[-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96]
CLO2.5	[0.001, 1.0], [0.002, 1.0], [0.003, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-9171.78 -9171.78 -9171.78] [-22172.58 -22172.58 -22172.58] [-35883.65 -35883.65 -35883.65]	[-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96]
CLO2.6	[0.003, 0.0], [0.004, 0.0], [0.005, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-24062.94 -24062.94 -24062.94] [-34236.97 -34236.97 -34236.97] [-42399.41 -42399.41 -42399.41]	[-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96]
CLO2.7	[0.001, 0.0], [0.002, 0.0], [0.003, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-9856.12 -9856.12 -9856.12] [-16971.52 -16971.52 -16971.52] [-24379.54 -24379.54 -24379.54]	[-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96]

Table A.8.: Simulation results of the second scenario for the cost function center lane offset

ID	u_veh1	u_veh2	J_veh1	J_veh2
CLO3.1	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-3145.77 -3145.77 -3145.77] [-2179.17 -2179.17 -2179.17] [-3702.41 -3702.41 -3702.41]	[-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96]
CLO3.2	[-0.001, 0.0], [-0.01, 0.0], [-0.03, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-6044.30 -6044.30 -6044.30] [-86577.94 -86577.94 -86577.94] [-245398.13 -245398.13 -245398.13]	[-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96]
CLO3.3	[-0.008, 0.0], [-0.009, 0.0], [-0.01, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-70071.77 -70071.77 -70071.77] [-79270.96 -79270.96 -79270.96] [-87596.99 -87596.99 -87596.99]	[-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96]
CLO3.4	[-0.004, 0.0], [-0.005, 0.0], [-0.006, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-34260.57 -34260.57 -34260.57] [-41164.66 -41164.66 -41164.66] [-53659.06 -53659.06 -53659.06]	[-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96]
CLO3.5	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0.001, 0.0], [-0.01, 0.0], [-0.03, 0.0]	[-3145.77 -3145.77 -3145.77] [-2179.17 -2179.17 -2179.17] [-3702.41 -3702.41 -3702.41]	[-3808.38 -57260.50 -162855.95] [-3808.38 -57260.50 -162855.95] [-3808.38 -57260.50 -162855.95]
CLO3.6	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0.008, 0.0], [-0.009, 0.0], [-0.01, 0.0]	[-3145.77 -3145.77 -3145.77] [-2179.17 -2179.17 -2179.17] [-3702.41 -3702.41 -3702.41]	[-45263.46 -52111.13 -58117.99] [-45263.46 -52111.13 -58117.99] [-45263.46 -52111.13 -58117.99]
CLO3.7	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0.004, 0.0], [-0.005, 0.0], [-0.006, 0.0]	[-3145.77 -3145.77 -3145.77] [-2179.17 -2179.17 -2179.17] [-3702.41 -3702.41 -3702.41]	[-22614.75 -29548.87 -35408.31] [-22614.75 -29548.87 -35408.31] [-22614.75 -29548.87 -35408.31]
CLO3.8	[-0.001, 1.0], [-0.01, 1.0], [-0.03, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-11849.26 -11849.26 -11849.26] [-117452.01 -117452.01 -117452.01] [-318495.70 -318495.70 -318495.70]	[-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96]
CLO3.9	[-0.008, 1.0], [-0.009, 1.0], [-0.01, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-90554.13 -90554.13 -90554.13] [-104234.04 -104234.04 -104234.04] [-115426.04 -115426.04 -115426.04]	[-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96]
CLO3.10	[-0.004, 1.0], [-0.005, 1.0], [-0.006, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-44976.28 -44976.28 -44976.28] [-57205.93 -57205.93 -57205.93] [-465955.17 -465955.17 -465955.17]	[-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96] [-2064.44 -1157.19 -2428.96]
CLO3.11	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0.001, 1.0], [-0.01, 1.0], [-0.03, 1.0]	[-3145.77 -3145.77 -3145.77] [-2179.17 -2179.17 -2179.17] [-3702.41 -3702.41 -3702.41]	[-8205.55 -78342.66 -220108.26] [-8205.55 -78342.66 -220108.26] [-8205.55 -78342.66 -220108.26]
CLO3.12	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0.008, 1.0], [-0.009, 1.0], [-0.01, 1.0]	[-3145.77 -3145.77 -3145.77] [-2179.17 -2179.17 -2179.17] [-3702.41 -3702.41 -3702.41]	[-62443.81 -68921.20 -82217.27] [-62443.81 -68921.20 -82217.27] [-62443.81 -68921.20 -82217.27]
CLO3.13	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0.004, 1.0], [-0.005, 1.0], [-0.006, 1.0]	[-3145.77 -3145.77 -3145.77] [-2179.17 -2179.17 -2179.17] [-3702.41 -3702.41 -3702.41]	[-31654.82 -37999.63 -48287.58] [-31654.82 -37999.63 -48287.58] [-31654.82 -37999.63 -48287.58]

Table A.9.: Simulation results of the third scenario for the cost function center lane offset

A.4. Simulation result for the cost function Euclidean distance

ID	u_veh1	u_veh2	J_veh1	J_veh2
ED1.1	[-0.01, 0.0], [-0.03, 0.0], [-0.05, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
ED1.2	[-0.03, 0.0], [-0.05, 0.0], [-0.07, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
ED1.3	[-0.01, 1.0], [-0.03, 1.0], [-0.05, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
ED1.4	[-0.03, 1.0], [-0.05, 1.0], [-0.07, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
ED1.5	[-0.005, 0.0], [-0.007, 0.0], [-0.009, 0.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
ED1.6	[-0.008, 0.0], [-0.009, 0.0], [-0.01, 0.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
ED1.7	[-0.02, 1.0], [-0.03, 1.0], [-0.04, 1.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
ED1.8	[-0.01, 1.0], [-0.02, 1.0], [-0.03, 1.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]

Table A.10.: Simulation results of the first scenario for the cost function Euclidean distance

ID	u_veh1	u_veh2	J_veh1	J_veh2
ED2.1	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
ED2.2	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[8535.19 4747.76 4709.12] [8274.63 5375.40 5282.33] [7771.43 2202.99 8323.14]	[5670.17 3143.70 3117.84] [5499.00 3557.64 3496.15] [5164.96 1453.53 5531.37]
ED2.3	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
ED2.4	[0.003, 1.0], [0.004, 1.0], [0.005, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
ED2.5	[0.001, 1.0], [0.002, 1.0], [0.003, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[8426.27 4924.67 7923.23] [-0. -0. -0.] [-0. -0. -0.]	[5597.17 3260.72 5265.73] [-0. -0. -0.] [-0. -0. -0.]

Table A.11.: Simulation results of the second scenario for the cost function Euclidean distance

ID	u_veh1	u_veh2	J_veh1	J_veh2
ED3.1	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
ED3.2	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
ED3.3	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
ED3.4	[0.008, 0.0], [0.009, 0.0], [0.01, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
ED3.5	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
ED3.6	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
ED3.7	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.008, 0.0], [0.009, 0.0], [0.01, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]

Table A.12.: Simulation results of the third scenario for the cost function Euclidean distance

A.5. Simulation result for the cost function obstacle distance

ID	u_veh1	u_veh2	J_veh1	J_veh2
OBS1.1	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [2467.44 2467.44 2467.44]	[3584.85 3618.86 3475.14] [3584.85 3618.86 3475.14] [3584.85 3618.86 3475.14]
OBS1.2	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [3061.35 3061.35 3061.35]	[3493.30 3532.46 3624.14] [3493.30 3532.46 3624.14] [3493.30 3532.46 3624.14]
OBS1.3	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.008, 0.0], [0.009, 0.0], [0.01, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [2467.44 2467.44 2467.44]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
OBS1.4	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.004, 0.0], [0.005, 0.0], [0.006, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [2467.44 2467.44 2467.44]	[1124.49 3724.25 -0.] [1124.49 3724.25 -0.] [1124.49 3724.25 -0.]
OBS1.5	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.008, 1.0], [0.009, 1.0], [0.01, 1.0]	[-0. -0. -0.] [-0. -0. -0.] [2467.44 2467.44 2467.44]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
OBS1.6	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.004, 1.0], [0.005, 1.0], [0.006, 1.0]	[-0. -0. -0.] [-0. -0. -0.] [2467.44 2467.44 2467.44]	[3951.48 2439.47 -0.] [3951.48 2439.47 -0.] [3951.48 2439.47 -0.]

Table A.13.: Simulation results of the first scenario for the cost function obstacle distance

ID	u_veh1	u_veh2	J_veh1	J_veh2
OBS2.1	[0.005, 0.0], [0.007, 0.0], [0.009, 0.0]	[0.015, 0.0], [0.016, 0.0], [0.017, 0.0]	[4509.06 4509.06 4509.06] [-0. -0. -0.] [-0. -0. -0.]	[2120.73 2118.73 -0.] [2120.73 2118.73 -0.] [2120.73 2118.73 -0.]
OBS2.2	[0.005, 0.0], [0.007, 0.0], [0.009, 0.0]	[0.015, 1.0], [0.016, 1.0], [0.017, 1.0]	[4509.06 4509.06 4509.06] [-0. -0. -0.] [-0. -0. -0.]	[4640.10 2369.82 -0.] [4640.10 2369.82 -0.] [4640.10 2369.82 -0.]
OBS2.3	[0.005, 1.0], [0.007, 1.0], [0.009, 1.0]	[0.015, 0.0], [0.016, 0.0], [0.017, 0.0]	[1897.22 1897.22 1897.22] [2709.46 2709.46 2709.46] [-0. -0. -0.]	[2136.44 -0. -0.] [2136.44 -0. -0.] [2136.44-0. -0.]
OBS2.4	[0.005, 1.0], [0.007, 1.0], [0.009, 1.0]	[0.015, 1.0], [0.016, 1.0], [0.017, 1.0]	[1897.22 1897.22 1897.22] [2709.46 2709.46 2709.46] [-0. -0. -0.]	[2415.00 2451.41 -0.] [2415.00 2451.41 -0.] [2415.00 2451.41 -0.]

Table A.14.: Simulation results of the second scenario for the cost function obstacle distance

ID	u_veh1	u_veh2	J_veh1	J_veh2
OBS3.1	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[4608.41 4608.41 4608.41] [4630.37 4630.37 4630.37] [4721.19 4721.19 4721.19]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
OBS3.2	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[4608.41 4608.41 4608.41] [4630.37 4630.37 4630.37] [4721.19 4721.19 4721.19]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
OBS3.3	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.07, 0.0], [0.08, 0.0], [0.09, 0.0]	[4608.41 4608.41 4608.41] [4630.37 4630.37 4630.37] [4721.19 4721.19 4721.19]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
OBS3.4	[0.005, 1.0], [0.007, 1.0], [0.009, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[1897.22 1897.22 1897.22] [2709.46 2709.46 2709.46] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
OBS3.5	[0.003, 1.0], [0.004, 1.0], [0.005, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[7065.98 7065.98 7065.98] [5114.12 5114.12 5114.12] [4916.97 4916.97 4916.97]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
OBS3.6	[0.005, 0.0], [0.007, 0.0], [0.009, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[4509.06 4509.06 4509.06] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
OBS3.7	[0.003, 0.0], [0.004, 0.0], [0.005, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[5890.48 5890.48 5890.48] [3483.33 3483.33 3483.33] [4756.14 4756.14 4756.14]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]

Table A.15.: Simulation results of the third scenario for the cost function obstacle distance

A.6. Simulation result for the cost function quadratic velocity

ID	u_veh1	u_veh2	J_veh1	J_veh2
VQ1.1	[0.0, 0.0], [0.0, 0.0], [0.00, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-9.79 -9.79 -9.79] [-9.79 -9.79 -9.79] [-9.79 -9.79 -9.79]
VQ1.2	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-9.79 -9.79 -9.79] [-9.79 -9.79 -9.79] [-9.79 -9.79 -9.79]
VQ1.3	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-9.79 -9.79 -9.79] [-9.79 -9.79 -9.79] [-9.79 -9.79 -9.79]
VQ1.4	[0.07, 0.0], [0.08, 0.0], [0.09, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-9.79 -9.79 -9.79] [-9.79 -9.79 -9.79] [-9.79 -9.79 -9.79]
VQ1.5	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-9.79 -9.79 -9.79] [-9.79 -9.79 -9.79] [-9.79 -9.79 -9.79]
VQ1.6	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-9.79 -9.79 -9.79] [-9.79 -9.79 -9.79] [-9.79 -9.79 -9.79]
VQ1.7	[0.07, 0.0], [0.08, 0.0], [0.09, 0.0]	[0.07, 0.0], [0.08, 0.0], [0.09, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-9.79 -9.79 -9.79] [-9.79 -9.79 -9.79] [-9.79 -9.79 -9.79]
VQ1.8	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-9.79 -9.79 -9.79] [-9.79 -9.79 -9.79] [-9.79 -9.79 -9.79]
VQ1.9	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[-28.70 -28.70 -28.70] [-0.96 -0.96 -0.96] [-0. -0. -0.]	[-2.03 -3.89 -9.79] [-2.03 -3.89 -9.79] [-2.03 -3.89 -9.79]
VQ1.10	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[-94.55 -94.55 -94.55] [-7.17 -7.17 -7.17] [-0. -0. -0.]	[-2.09 -3.90 -12.95] [-2.09 -3.90 -12.95] [-2.09 -3.90 -12.95]
VQ1.11	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-97.96 -97.96 -97.96] [-97.97 -97.97 -97.97] [-97.97 -97.97 -97.97]	[-0.46 -0.46 -0.46] [-0.46 -0.46 -0.46] [-0.46 -0.46 -0.46]
VQ1.12	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0.46 -0.46 -0.46] [-0.46 -0.46 -0.46] [-0.46 -0.46 -0.46]	[-64.33 -64.33 -64.33] [-64.33 -64.33 -64.33] [-64.33 -64.33 -64.33]

Table A.16.: Simulation results of the first scenario for the cost function quadratic velocity

ID	u_veh1	u_veh2	J_veh1	J_veh2
VQ2.1	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-9.79 -9.79 -9.79] [-9.79 -9.79 -9.79] [-9.79 -9.79 -9.79]
VQ2.2	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[-28.70 -28.70 -28.70] [-0.96 -0.96 -0.96] [-0. -0. -0.]	[-2.03 -3.89 -9.79] [-2.03 -3.89 -9.79] [-2.03 -3.89 -9.79]
VQ2.3	[0.0, 1.0], [0.0, 0.5], [0.00, 0.0]	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[-94.55 -94.55 -94.55] [-7.17 -7.17 -7.17] [-0. -0. -0.]	[-2.09 -3.90 -12.95] [-2.09 -3.90 -12.95] [-2.09 -3.90 -12.95]
VQ2.4	[0.0, 0.0], [0.0, 0.0], [0.00, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-97.96 -97.96 -97.96] [-97.97 -97.97 -97.97] [-97.97 -97.97 -97.97]	[-0.46 -0.46 -0.46] [-0.46 -0.46 -0.46] [-0.46 -0.46 -0.46]
VQ2.5	[0.0, 0.0], [0.0, 0.0], [0.00, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0.46 -0.46 -0.46] [-0.46 -0.46 -0.46] [-0.46 -0.46 -0.46]	[-64.33 -64.33 -64.33] [-64.33 -64.33 -64.33] [-64.33 -64.33 -64.33]

Table A.17.: Simulation results of the second scenario for the cost function quadratic velocity

ID	u_veh1	u_veh2	J_veh1	J_veh2
VQ3.1	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-9.79 -9.79 -9.79] [-9.79 -9.79 -9.79] [-9.79 -9.79 -9.79]
VQ3.2	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[-28.70 -28.70 -28.70] [-0.96 -0.96 -0.96] [-0. -0. -0.]	[-2.03 -3.89 -9.79] [-2.03 -3.89 -9.79] [-2.03 -3.89 -9.79]
VQ3.3	[0.0, 1.0], [0.0, 0.5], [0.00, 0.0]	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[-94.55 -94.55 -94.55] [-7.17 -7.17 -7.17] [-0. -0. -0.]	[-2.09 -3.90 -12.95] [-2.09 -3.90 -12.95] [-2.09 -3.90 -12.95]
VQ3.4	[0.0, 0.0], [0.0, 0.0], [0.00, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-97.96 -97.96 -97.96] [-97.97 -97.97 -97.97] [-97.97 -97.97 -97.97]	[-0.46 -0.46 -0.46] [-0.46 -0.46 -0.46] [-0.46 -0.46 -0.46]
VQ3.5	[0.0, 0.0], [0.0, 0.0], [0.00, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0.46 -0.46 -0.46] [-0.46 -0.46 -0.46] [-0.46 -0.46 -0.46]	[-64.33 -64.33 -64.33] [-64.33 -64.33 -64.33] [-64.33 -64.33 -64.33]

Table A.18.: Simulation results of the third scenario for the cost function quadratic velocity

A.7. Simulation result for the cost function steering angle

ID	u_veh1	u_veh2	J_veh1	J_veh2
ST1.1	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-10.52 -10.52 -10.52] [-13.32 -13.32 -13.32] [-14.51 -14.51 -14.51]	[-9.76 -10.44 -11.29] [-9.76 -10.44 -11.29] [-9.76 -10.44 -11.29]
ST1.2	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-15.03 -15.03 -15.03] [-13.47 -13.47 -13.47] [-12.50 -12.50 -12.50]	[-11.35 -10.82 -7.86] [-11.35 -10.82 -7.86] [-11.35 -10.82 -7.86]
ST1.3	[-0.01, 0.0], [-0.03, 0.0], [-0.05, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-360.59 -360.59 -360.59] [-1078.60 -1078.60 -1078.60] [-1793.01 -1793.01 -1793.01]	[-10.18 -9.97 -10.75] [-10.18 -9.97 -10.75] [-10.18 -9.97 -10.75]
ST1.4	[-0.001, 0.0], [-0.005, 0.0], [-0.009, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-35.30 -35.30 -35.30] [-176.84 -176.84 -176.84] [-323.46 -323.46 -323.46]	[-10.31 -10.05 -10.22] [-10.31 -10.05 -10.22] [-10.31 -10.05 -10.22]
ST1.5	[-0.01, 1.0], [-0.03, 1.0], [-0.05, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-410.54 -410.54 -410.54] [-1244.71 -1244.71 -1244.71] [-2067.92 -2067.92 -2067.92]	[-9.66 -11.56 -9.53] [-9.66 -11.56 -9.53] [-9.66 -11.56 -9.53]
ST1.6	[-0.001, 1.0], [-0.005, 1.0], [-0.009, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-43.37 -43.37 -43.37] [-204.99 -204.99 -204.99] [-376.70 -376.70 -376.70]	[-7.89 -9.61 -8.47] [-7.89 -9.61 -8.47] [-7.89 -9.61 -8.47]
ST1.7	[-0.01, 0.0], [-0.03, 0.0], [-0.05, 0.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-360.59 -360.59 -360.59] [-1078.60 -1078.60 -1078.60] [-1793.01 -1793.01 -1793.01]	[-11.81 -9.55 -8.18] [-11.81 -9.55 -8.18] [-11.81 -9.55 -8.18]
ST1.8	[-0.001, 0.0], [-0.005, 0.0], [-0.009, 0.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-35.30 -35.30 -35.30] [-176.84 -176.84 -176.84] [-323.46 -323.46 -323.46]	[-15.00 -11.71 -10.49] [-15.00 -11.71 -10.49] [-15.00 -11.71 -10.49]
ST1.9	[-0.01, 1.0], [-0.03, 1.0], [-0.05, 1.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-410.54 -410.54 -410.54] [-1244.71 -1244.71 -1244.71] [-2067.92 -2067.92 -2067.92]	[-11.67 -13.27 -10.50] [-11.67 -13.27 -10.50] [-11.67 -13.27 -10.50]
ST1.10	[-0.001, 1.0], [-0.005, 1.0], [-0.009, 1.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-43.37 -43.37 -43.37] [-204.99 -204.99 -204.99] [-376.70 -376.70 -376.70]	[-10.85 -11.23 -12.04] [-10.85 -11.23 -12.04] [-10.85 -11.23 -12.04]

Table A.19.: Simulation results of the first scenario for the cost function steering angle

ID	u_veh1	u_veh2	J_veh1	J_veh2
ST2.1	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-10.52 -10.52 -10.52] [-13.32 -13.32 -13.32] [-14.51 -14.51 -14.51]	[-9.76 -10.44 -11.29] [-9.76 -10.44 -11.29] [-9.76 -10.44 -11.29]
ST2.2	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-15.03 -15.03 -15.03] [-13.47 -13.47 -13.47] [-12.50 -12.50 -12.50]	[-11.35 -10.82 -7.86] [-11.35 -10.82 -7.86] [-11.35 -10.82 -7.86]
ST2.3	[0.003, 0.0], [0.004, 0.0], [0.005, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-108.07 -108.07 -108.07] [-142.10 -142.10 -142.10] [-180.74 -180.74 -180.74]	[-10.93 -11.00 -12.27] [-10.93 -11.00 -12.27] [-10.93 -11.00 -12.27]
ST2.4	[0.001, 0.0], [0.002, 0.0], [0.003, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-38.83 -38.83 -38.83] [-76.94 -76.94 -76.94] [-110.04 -110.04 -110.04]	[-9.72 -13.50 -10.07] [-9.72 -13.50 -10.07] [-9.72 -13.50 -10.07]
ST2.5	[0.003, 1.0], [0.004, 1.0], [0.005, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-122.16 -122.16 -122.16] [-166.16 -166.16 -166.16] [-204.35 -204.35 -204.35]	[-10.68 -10.07 -10.72] [-10.68 -10.07 -10.72] [-10.68 -10.07 -10.72]
ST2.6	[0.001, 1.0], [0.002, 1.0], [0.003, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-42.60 -42.60 -42.60] [-82.24 -82.24 -82.24] [-117.96 -117.96 -117.96]	[-10.93 -9.99 -10.56] [-10.93 -9.99 -10.56] [-10.93 -9.99 -10.56]

Table A.20.: Simulation results of the second scenario for the cost function steering angle

ID	u_veh1	u_veh2	J_veh1	J_veh2
ST3.1	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-1436.76 -1436.76 -1436.76] [-1791.98 -1791.98 -1791.98] [-2153.84 -2153.84 -2153.84]	[-9.19 -10.83 -10.13] [-9.19 -10.83 -10.13] [-9.19 -10.83 -10.13]
ST3.2	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-358.15 -358.15 -358.15] [-719.42 -719.42 -719.42] [-1074.40 -1074.40 -1074.40]	[-10.52 -11.83 -8.65] [-10.52 -11.83 -8.65] [-10.52 -11.83 -8.65]
ST3.3	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[-10.52 -10.52 -10.52] [-10.99 -10.99 -10.99] [-13.00 -13.00 -13.00]	[-1162.80 -1455.47 -1740.59] [-1162.80 -1455.47 -1740.59] [-1162.80 -1455.47 -1740.59]
ST3.4	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[-10.52 -10.52 -10.52] [-10.99 -10.99 -10.99] [-13.00 -13.00 -13.00]	[-289.86 -581.19 -872.09] [-289.86 -581.19 -872.09] [-289.86 -581.19 -872.09]
ST3.5	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[-358.15 -358.15 -358.15] [-715.10 -715.10 -715.10] [-1081.75 -1081.75 -1081.75]	[-289.71 -583.73 -872.15] [-289.71 -583.73 -872.15] [-289.71 -583.73 -872.15]
ST3.6	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[-1436.76 -1436.76 -1436.76] [-1792.08 -1792.08 -1792.08] [-2154.63 -2154.63 -2154.63]	[-1161.17 -1451.92 -1743.97] [-1161.17 -1451.92 -1743.97] [-1161.17 -1451.92 -1743.97]
ST3.7	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[-358.15 -358.15 -358.15] [-715.10 -715.10 -715.10] [-1081.75 -1081.75 -1081.75]	[-1164.36 -1456.85 -1746.94] [-1164.36 -1456.85 -1746.94] [-1164.36 -1456.85 -1746.94]
ST3.8	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[-1436.76 -1436.76 -1436.76] [-1792.08 -1792.08 -1792.08] [-2154.63 -2154.63 -2154.63]	[-292.20 -581.03 -874.87] [-292.20 -581.03 -874.87] [-292.20 -581.03 -874.87]
ST3.9	[0.01, 1.0], [0.02, 1.0], [0.03, 1.0]	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[-417.58 -417.58 -417.58] [-826.34 -826.34 -826.34] [-1240.23 -1240.23 -1240.23]	[-291.83 -582.56 -872.50] [-291.83 -582.56 -872.50] [-291.83 -582.56 -872.50]
ST3.10	[0.04, 1.0], [0.05, 1.0], [0.06, 1.0]	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[-1658.36 -1658.36 -1658.36] [-2064.95 -2064.95 -2064.95] [-2478.65 -2478.65 -2478.65]	[-1166.40 -1451.62 -1744.01] [-1166.40 -1451.62 -1744.01] [-1166.40 -1451.62 -1744.01]
ST3.11	[0.01, 1.0], [0.02, 1.0], [0.03, 1.0]	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[-417.58 -417.58 -417.58] [-826.34 -826.34 -826.34] [-1240.23 -1240.23 -1240.23]	[-1160.99 -1449.24 -1742.40] [-1160.99 -1449.24 -1742.40] [-1160.99 -1449.24 -1742.40]
ST3.12	[0.04, 1.0], [0.05, 1.0], [0.06, 1.0]	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[-1658.36 -1658.36 -1658.36] [-2064.95 -2064.95 -2064.95] [-2478.65 -2478.65 -2478.65]	[-291.15 -580.14 -873.10] [-291.15 -580.14 -873.10] [-291.15 -580.14 -873.10]
ST3.13	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[0.01, 1.0], [0.02, 1.0], [0.03, 1.0]	[-358.15 -358.15 -358.15] [-715.10 -715.10 -715.10] [-1081.75 -1081.75 -1081.75]	[-341.06 -677.68 -1020.81] [-341.06 -677.68 -1020.81] [-341.06 -677.68 -1020.81]
ST3.14	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[0.04, 1.0], [0.05, 1.0], [0.06, 1.0]	[-1436.76 -1436.76 -1436.76] [-1792.08 -1792.08 -1792.08] [-2154.63 -2154.63 -2154.63]	[-1361.72 -1703.25 -2040.97] [-1361.72 -1703.25 -2040.97] [-1361.72 -1703.25 -2040.97]
ST3.15	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[0.04, 1.0], [0.05, 1.0], [0.06, 1.0]	[-358.15 -358.15 -358.15] [-715.10 -715.10 -715.10] [-1081.75 -1081.75 -1081.75]	[-1364.36 -1698.58 -2040.37] [-1364.36 -1698.58 -2040.37] [-1364.36 -1698.58 -2040.37]
ST3.16	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[0.01, 1.0], [0.02, 1.0], [0.03, 1.0]	[-1436.76 -1436.76 -1436.76] [-1792.08 -1792.08 -1792.08] [-2154.63 -2154.63 -2154.63]	[-338.15 -676.85 -1021.70] [-338.15 -676.85 -1021.70] [-338.15 -676.85 -1021.70]
ST3.17	[0.01, 1.0], [0.02, 1.0], [0.03, 1.0]	[0.01, 1.0], [0.02, 1.0], [0.03, 1.0]	[-417.58 -417.58 -417.58] [-826.34 -826.34 -826.34] [-1240.23 -1240.23 -1240.23]	[-339.90 -679.73 -1017.19] [-339.90 -679.73 -1017.19] [-339.90 -679.73 -1017.19]
ST3.18	[0.04, 1.0], [0.05, 1.0], [0.06, 1.0]	[0.04, 1.0], [0.05, 1.0], [0.06, 1.0]	[-1658.36 -1658.36 -1658.36] [-2064.95 -2064.95 -2064.95] [-2478.65 -2478.65 -2478.65]	[-1362.81 -1701.62 -2040.41] [-1362.81 -1701.62 -2040.41] [-1362.81 -1701.62 -2040.41]
ST3.19	[0.01, 1.0], [0.02, 1.0], [0.03, 1.0]	[0.04, 1.0], [0.05, 1.0], [0.06, 1.0]	[-417.58 -417.58 -417.58] [-826.34 -826.34 -826.34] [-1240.23 -1240.23 -1240.23]	[-1360.07 -1700.22 -2042.50] [-1360.07 -1700.22 -2042.50] [-1360.07 -1700.22 -2042.50]

Table A.21.: Simulation results of the third scenario for the cost function steering angle

A.8. Simulation results for the cost function acceleration work

ID	u_veh1	u_veh2	J_veh1	J_veh2
AC1.1	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
AC1.2	[-0.004, 0.0], [-0.005.0, 0.0], [-0.006, 0.0]	[-0.004, 0.0], [-0.005.0, 0.0], [-0.006, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
AC1.3	[-0.01, 0.0], [-0.02, 0.0], [-0.03, 0.0]	[-0.01, 0.0], [-0.02, 0.0], [-0.03, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
AC1.4	[-0.04, 0.0], [-0.05, 0.0], [-0.06, 0.0]	[-0.04, 0.0], [-0.05, 0.0], [-0.06, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
AC1.5	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-0.03 -0.03 -0.03] [-0.03 -0.03 -0.03] [-0.03 -0.03 -0.03]	[-0.03 -0.03 -0.03] [-0.03 -0.03 -0.03] [-0.03 -0.03 -0.03]
AC1.6	[0.0, 0.5], [0.0, 0.5], [0.0, 0.5]	[0.0, 0.5], [0.0, 0.5], [0.0, 0.5]	[-0.009 -0.009 -0.009] [-0.009 -0.009 -0.009] [-0.009 -0.009 -0.009]	[-0.009 -0.009 -0.009] [-0.009 -0.009 -0.009] [-0.009 -0.009 -0.009]
AC1.7	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[-0.03 -0.03 -0.03] [-0.009 -0.009 -0.009] [-0. -0. -0.]	[-0.03 -0.03 -0.03] [-0.009 -0.009 -0.009] [-0. -0. -0.]
AC1.8	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[-0.09 -0.09 -0.09] [-0.02 -0.02 -0.02] [-0. -0. -0.]	[-0.03 -0.03 -0.03] [-0.009 -0.009 -0.009] [-0. -0. -0.]

Table A.22.: Simulation results of the first scenario for the cost function acceleration work

ID	u_veh1	u_veh2	J_veh1	J_veh2
AC2.1	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-0.03 -0.03 -0.03] [-0.03 -0.03 -0.03] [-0.03 -0.03 -0.03]	[-0.03 -0.03 -0.03] [-0.03 -0.03 -0.03] [-0.03 -0.03 -0.03]
AC2.2	[0.0, 0.5], [0.0, 0.5], [0.0, 0.5]	[0.0, 0.5], [0.0, 0.5], [0.0, 0.5]	[-0.009 -0.009 -0.009] [-0.009 -0.009 -0.009] [-0.009 -0.009 -0.009]	[-0.009 -0.009 -0.009] [-0.009 -0.009 -0.009] [-0.009 -0.009 -0.009]
AC2.3	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[-0.03 -0.03 -0.03] [-0.009 -0.009 -0.009] [-0. -0. -0.]	[-0.03 -0.03 -0.03] [-0.009 -0.009 -0.009] [-0. -0. -0.]
AC2.4	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[-0.09 -0.09 -0.09] [-0.02 -0.02 -0.02] [-0. -0. -0.]	[-0.03 -0.03 -0.03] [-0.009 -0.009 -0.009] [-0. -0. -0.]

Table A.23.: Simulation results of the second scenario for the cost function acceleration work

ID	u_veh1	u_veh2	J_veh1	J_veh2
AC3.1	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-0.03 -0.03 -0.03] [-0.03 -0.03 -0.03] [-0.03 -0.03 -0.03]	[-0.03 -0.03 -0.03] [-0.03 -0.03 -0.03] [-0.03 -0.03 -0.03]
AC3.2	[0.0, 0.5], [0.0, 0.5], [0.0, 0.5]	[0.0, 0.5], [0.0, 0.5], [0.0, 0.5]	[-0.009 -0.009 -0.009] [-0.009 -0.009 -0.009] [-0.009 -0.009 -0.009]	[-0.009 -0.009 -0.009] [-0.009 -0.009 -0.009] [-0.009 -0.009 -0.009]
AC3.3	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[-0.03 -0.03 -0.03] [-0.009 -0.009 -0.009] [-0. -0. -0.]	[-0.03 -0.03 -0.03] [-0.009 -0.009 -0.009] [-0. -0. -0.]
AC3.4	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[-0.09 -0.09 -0.09] [-0.02 -0.02 -0.02] [-0. -0. -0.]	[-0.03 -0.03 -0.03] [-0.009 -0.009 -0.009] [-0. -0. -0.]

Table A.24.: Simulation results of the third scenario for the cost function acceleration work

B. Simulation results for the optimized cost functions

B.1. Simulation results for the optimized cost function collision energy

ID	u_veh1	u_veh2	J_veh1	J_veh2
CEopt1.1	[-0.004, 0.0], [-0.005, 0.0], [-0.006, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
CEopt1.2	[-0.007, 0.0], [-0.008, 0.0], [-0.009, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
CEopt1.3	[-0.004, 0.0], [-0.006, 0.0], [-0.008, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
CEopt1.4	[-0.004, 1.0], [-0.005, 1.0], [-0.006, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
CEopt1.5	[-0.007, 1.0], [-0.008, 1.0], [-0.009, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
CEopt1.6	[-0.004, 1.0], [-0.006, 1.0], [-0.008, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]

Table B.1.: Simulation results of the first scenario for the optimized cost function collision energy

ID	u_veh1	u_veh2	J_veh1	J_veh2
CEopt2.1	[-0.0, 0.0], [0.0, 0.0], [0.00, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
CEopt2.2	[-0.0, 1.0], [0.0, 1.0], [0.00, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
CEopt2.3	[-0.0, 0.0], [0.0, 0.0], [0.00, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]

Table B.2.: Simulation results of the second scenario for the optimized cost function collision energy

ID	u_veh1	u_veh2	J_veh1	J_veh2
CEopt3.1	[0.0, 0.0], [0.0, 0.0], [0.00, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
CEopt3.2	[0.07, 0.0], [0.08, 0.0], [0.09, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
CEopt3.3	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
CEopt3.4	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
CEopt3.5	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
CEopt3.6	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
CEopt3.7	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
CEopt3.8	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
CEopt3.9	[0.01, 1.0], [0.02, 1.0], [0.03, 1.0]	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
CEopt3.11	[0.04, 1.0], [0.05, 1.0], [0.06, 1.0]	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]

Table B.3.: Simulation results of the third scenario for the optimized cost function collision energy

B.2. Simulation results for the optimized cost function Time-to-Collision

ID	u_veh1	u_veh2	J_veh1	J_veh2
TTCopt1.1	[-0.01, 0.0], [-0.02, 0.0], [-0.03, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [4480.44 4480.44 4480.44]	[-0. -0. -0.] [-0. -0. -0.] [3629.21 3629.21 3629.21]
TTCopt1.2	[-0.03, 0.0], [-0.04, 0.0], [-0.05, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[4480.19 4480.19 4480.19] [5961.35 5961.35 5961.35] [5677.13 5677.13 5677.13]	[3629.21 3629.21 3629.21] [4829.17 4829.17 4829.17] [4599.02 4599.02 4599.02]
TTCopt1.3	[-0.01, 1.0], [-0.02, 1.0], [-0.03, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
TTCopt1.4	[-0.03, 1.0], [-0.04, 1.0], [-0.05, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [2973.32 2973.32 2973.32] [3544.73 3544.73 3544.73]	[-0. -0. -0.] [2252.55 2252.55 2252.55] [2695.57 2695.57 2695.57]
TTCopt1.5	[-0.01, 0.0], [-0.02, 0.0], [-0.03, 0.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[14871.30 14871.30 14871.30] [14405.65 14405.65 14405.65] [11908.54 11908.54 11908.54]	[13820.35 13820.35 13820.35] [12951.51 12951.51 12951.51] [10533.16 10533.16 10533.16]
TTCopt1.6	[-0.03, 0.0], [-0.04, 0.0], [-0.05, 0.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[11908.54 11908.54 11908.54] [10872.27 10872.27 10872.27] [10031.67 10031.67 10031.67]	[10533.16 10533.16 10533.16] [9541.18 9541.18 9541.18] [8728.82 8728.82 8728.82]
TTCopt1.7	[-0.01, 1.0], [-0.02, 1.0], [-0.03, 1.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-0. -0. -0.] [-0. -0. -0.] [4777.03 4777.03 4777.03]	[-0. -0. -0.] [-0. -0. -0.] [3902.45 3902.45 3902.45]
TTCopt1.8	[-0.03, 1.0], [-0.04, 1.0], [-0.05, 1.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[4777.03 4777.03 4777.03] [6350.77 6350.77 6350.77] [6001.53 6001.53 6001.53]	[3902.45 3902.45 3902.45] [5184.31 5184.31 5184.31] [4895.35 4895.35 4895.35]

Table B.4.: Simulation results of the first scenario for the optimized cost function Time-to-Collision

ID	u_veh1	u_veh2	J_veh1	J_veh2
TTCopt2.1	[-0.0, 0.0], [0.0, 0.0], [0.00, 0.0]	[-0.000, 0.0], [0.0, 0.0], [0.000, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
TTCopt2.2	[-0.0, 1.0], [0.0, 1.0], [0.00, 1.0]	[-0.000, 0.0], [0.0, 0.0], [0.000, 0.0]	[882.35 882.35 882.35] [882.35 882.35 882.35] [882.35 882.35 882.35]	[584.26 584.26 584.26] [584.26 584.26 584.26] [584.26 584.26 584.26]
TTCopt2.3	[-0.0, 1.0], [0.0, 1.0], [0.00, 1.0]	[-0.0, 1.0], [0.0, 1.0], [0.00, 1.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
TTCopt2.4	[0.003, 1.0], [0.004, 1.0], [0.005, 1.0]	[-0.000, 0.0], [0.0, 0.0], [0.000, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
TTCopt2.5	[0.001, 1.0], [0.002, 1.0], [0.003, 1.0]	[-0.000, 0.0], [0.0, 0.0], [0.000, 0.0]	[955.45 955.45 955.45] [-0. -0. -0.] [-0. -0. -0.]	[632.62 632.62 632.62] [-0. -0. -0.] [-0. -0. -0.]

Table B.5.: Simulation results of the second scenario for the optimized cost function Time-to-Collision

ID	u_veh1	u_veh2	J_veh1	J_veh2
TTCopt3.1	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0.000, 0.0], [0.0, 0.0], [0.000, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
TTCopt3.2	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[-0.000, 0.0], [0.0, 0.0], [0.000, 0.0]	[1494.73 1494.73 1494.73] [1430.31 1430.31 1430.31] [1504.10 1504.10 1504.10]	[1210.73 1210.73 1210.73] [1158.55 1158.55 1158.55] [1218.32 1218.32 1218.32]
TTCopt3.3	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[-0.000, 0.0], [0.0, 0.0], [0.000, 0.0]	[-0. -0. -0.] [889.65 889.65 889.65] [1681.93 1681.93 1681.93]	[-0. -0. -0.] [720.62 720.62 720.62] [1362.36 1362.36 1362.36]
TTCopt3.4	[0.008, 0.0], [0.009, 0.0], [0.01, 0.0]	[-0.000, 0.0], [0.0, 0.0], [0.000, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
TTCopt3.5	[-0.000, 0.0], [0.0, 0.0], [0.000, 0.0]	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[1541.95 1448.83 1443.82] [1541.95 1448.83 1443.82] [1541.95 1448.83 1443.82]	[1248.98 1173.55 1169.49] [1248.98 1173.55 1169.49] [1248.98 1173.55 1169.49]
TTCopt3.6	[-0.000, 0.0], [0.0, 0.0], [0.000, 0.0]	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[-0. -0. 1708.66] [-0. -0. 1708.66] [-0. -0. 1708.66]	[-0. -0. 1384.02] [-0. -0. 1384.02] [-0. -0. 1384.02]
TTCopt3.7	[-0.000, 0.0], [0.0, 0.0], [0.000, 0.0]	[0.008, 0.0], [0.009, 0.0], [0.01, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
TTCopt3.8	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[912.56 798.93 1525.47] [775.76 1568.33 1460.32] [1533.42 1460.76 1530.18]	[739.17 647.14 1235.63] [628.36 1270.34 1182.86] [1242.07 1183.21 1239.44]
TTCopt3.9	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[812.50 931.57 -0.] [-0. -0. -0.] [-0. -0. -0.]	[658.13 754.57 -0.] [-0. -0. -0.] [-0. -0. -0.]
TTCopt3.10	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[1456.23 1475.60 1580.21] [1504.09 685.39 780.11] [693.15 789.98 909.17]	[1179.55 1195.24 1279.97] [1218.31 555.16 631.89] [561.45 639.88 736.43]
TTCopt3.11	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[1446.69 1487.27 715.30] [1496.26 1641.27 846.37] [1669.54 836.80 -0.]	[1171.82 1204.69 579.39] [1211.97 1329.43 685.56] [1352.33 677.80 -0.]
TTCopt3.12	[0.01, 1.0], [0.02, 1.0], [0.03, 1.0]	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[631.23 530.10 395.66] [503.04 419.50 341.76] [406.33 358.21 366.39]	[473.08 397.29 296.53] [377.01 314.40 256.13] [304.53 268.47 274.59]
TTCopt3.13	[0.04, 1.0], [0.05, 1.0], [0.06, 1.0]	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[547.82 652.49 -0.] [686.70 -0. -0.] [-0. -0. -0.]	[410.57 489.02 -0.] [514.66 -0. -0.] [-0. -0. -0.]
TTCopt3.14	[0.01, 1.0], [0.02, 1.0], [0.03, 1.0]	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[356.47 363.37 414.02] [357.21 415.57 502.02] [430.53 520.29 624.60]	[267.16 272.33 310.30] [267.71 311.46 376.24] [322.66 389.94 468.12]
TTCopt3.15	[0.04, 1.0], [0.05, 1.0], [0.06, 1.0]	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[366.89 371.40 456.84] [401.87 452.46 583.05] [495.60 573.07 726.23]	[274.97 278.35 342.38] [301.19 339.10 436.98] [371.44 429.50 544.28]
TTCopt3.16	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[0.01, 1.0], [0.02, 1.0], [0.03, 1.0]	[1350.79 1123.41 828.47] [1088.30 898.55 729.04] [884.67 768.69 783.71]	[1192.54 991.80 731.41] [960.81 793.28 643.63] [781.03 678.63 691.89]
TTCopt3.17	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[0.04, 1.0], [0.05, 1.0], [0.06, 1.0]	[1172.54 1410.60 -0.] [1454.7334678 -0. -0.] [-0. -0. -0.]	[1035.18 1245.35 -0.] [1284.30 -0. -0.] [-0. -0. -0.]
TTCopt3.18	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[0.04, 1.0], [0.05, 1.0], [0.06, 1.0]	[755.74 794.51 930.81] [775.51 919.76 1124.52] [932.50 1139.78 1379.14]	[667.20 701.43 821.76] [684.66 812.00 992.78] [823.25 1006.26 1217.57]
TTCopt3.19	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[0.01, 1.0], [0.02, 1.0], [0.03, 1.0]	[787.08 782.57 966.45] [833.74 933.72 1220.73] [1004.49 1169.66 1509.59]	[694.87 690.89 853.22] [736.07 824.33 1077.72] [886.81 1032.63 1332.74]

Table B.6.: Simulation results of the third scenario for the optimized cost function Time-to-Collision

B.3. Simulation results for the optimized cost function center lane offset

ID	u_veh1	u_veh2	J_veh1	J_veh2
CLOopt1.1	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-97921.75 -97921.75 -97921.75] [-97921.75 -97921.75 -97921.75] [-97921.75 -97921.75 -97921.75]	[-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10]
CLOopt1.2	[-0.01, 0.0], [-0.03, 0.0], [-0.05, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-94117.10 -94117.10 -94117.10] [-228131.15 -228131.15 -228131.15] [-320768.35 -320768.35 -320768.35]	[-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10]
CLOopt1.3	[-0.008, 0.0], [-0.009, 0.0], [-0.01, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-81873.20 -81873.20 -81873.20] [-88507.41 -88507.41 -88507.41] [-94117.10 -94117.10 -94117.10]	[-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10]
CLOopt1.4	[-0.004, 0.0], [-0.005, 0.0], [-0.006, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-64954.59 -64954.59 -64954.59] [-67849.12 -67849.12 -67849.12] [-71807.22 -71807.22 -71807.22]	[-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10]
CLOopt1.5	[-0.001, 1.0], [-0.01, 1.0], [-0.03, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-85866.01 -85866.01 -85866.01] [-119237.06 -119237.06 -119237.06] [-295993.80 -295993.80 -295993.80]	[-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10]
CLOopt1.6	[-0.008, 1.0], [-0.009, 1.0], [-0.01, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-101972.27 -101972.27 -101972.27] [-109793.52 -109793.52 -109793.52] [-119237.06 -119237.06 -119237.06]	[-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10]
CLOopt1.7	[-0.004, 1.0], [-0.005, 1.0], [-0.006, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-75100.92 -75100.92 -75100.92] [-79211.53 -79211.53 -79211.53] [-85660.82 -85660.82 -85660.82]	[-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10]
CLOopt1.8	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-111338.71 -111338.71 -111338.71] [-111338.71 -111338.71 -111338.71] [-111338.71 -111338.71 -111338.71]	[-10704.91 -10704.91 -10704.91] [-10704.91 -10704.91 -10704.91] [-10704.91 -10704.91 -10704.91]

Table B.7.: Simulation results of the first scenario for the optimized cost function center lane offset

ID	u_veh1	u_veh2	J_veh1	J_veh2
CLOopt2.1	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-12434.56 -12434.56 -12434.56] [-12434.56 -12434.56 -12434.56] [-12434.56 -12434.56 -12434.56]	[-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10]
CLOopt2.2	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-16618.23 -16618.23 -16618.23] [-16618.23 -16618.23 -16618.23] [-16618.23 -16618.23 -16618.23]	[-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10]
CLOopt2.3	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-16618.23 -16618.23 -16618.23] [-16618.23 -16618.23 -16618.23] [-16618.23 -16618.23 -16618.23]	[-10704.91 -10704.91 -10704.91] [-10704.91 -10704.91 -10704.91] [-10704.91 -10704.91 -10704.91]
CLOopt2.4	[0.003, 1.0], [0.004, 1.0], [0.005, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-41178.57 -41178.57 -41178.57] [-55692.21 -55692.21 -55692.21] [-68840.58 -68840.58 -68840.58]	[-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10]
CLOopt2.5	[0.001, 1.0], [0.002, 1.0], [0.003, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-18456.53 -18456.53 -18456.53] [-41178.57 -41178.57 -41178.57]	[-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10]
CLOopt2.6	[0.003, 0.0], [0.004, 0.0], [0.005, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-28978.06 -28978.06 -28978.06] [-40632.81 -40632.81 -40632.81] [-51845.55 -51845.55 -51845.55]	[-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10]
CLOopt2.7	[0.001, 0.0], [0.002, 0.0], [0.003, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-11279.98 -11279.98 -11279.98] [-28978.06 -28978.06 -28978.06]	[-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10]

Table B.8.: Simulation results of the second scenario for the optimized cost function center lane offset

ID	u_veh1	u_veh2	J_veh1	J_veh2
CLOopt3.1	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-12434.56 -12434.56 -12434.56] [-12434.56 -12434.56 -12434.56] [-12434.56 -12434.56 -12434.56]	[-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10]
CLOopt3.2	[-0.001, 0.0], [-0.01, 0.0], [-0.03, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-21581.66 -21581.66 -21581.66] [-101845.11 -101845.11 -101845.11] [-257044.41 -257044.41 -257044.41]	[-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10]
CLOopt3.3	[-0.008, 0.0], [-0.009, 0.0], [-0.01, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-84562.26 -84562.26 -84562.26] [-387524.20 -387524.20 -387524.20] [-101845.11 -101845.11 -101845.11]	[-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10]
CLOopt3.4	[-0.004, 0.0], [-0.005, 0.0], [-0.006, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-48869.83 -48869.83 -48869.83] [-57937.12 -57937.12 -57937.12] [-66686.04 -66686.04 -66686.04]	[-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10]
CLOopt3.5	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0.001, 0.0], [-0.01, 0.0], [-0.03, 0.0]	[-12434.56 -12434.56 -12434.56] [-12434.56 -12434.56 -12434.56] [-12434.56 -12434.56 -12434.56]	[-13944.28 -66654.50 -171685.55] [-13944.28 -66654.50 -171685.55] [-13944.28 -66654.50 -171685.55]
CLOopt3.6	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0.008, 0.0], [-0.009, 0.0], [-0.01, 0.0]	[-12434.56 -12434.56 -12434.56] [-12434.56 -12434.56 -12434.56] [-12434.56 -12434.56 -12434.56]	[-55274.67 -60982.88 -66654.50] [-55274.67 -60982.88 -66654.50] [-55274.67 -60982.88 -66654.50]
CLOopt3.7	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0.004, 0.0], [-0.005, 0.0], [-0.006, 0.0]	[-12434.56 -12434.56 -12434.56] [-12434.56 -12434.56 -12434.56] [-12434.56 -12434.56 -12434.56]	[-31769.41 -37834.54 -43575.26] [-31769.41 -37834.54 -43575.26] [-31769.41 -37834.54 -43575.26]
CLOopt3.8	[-0.001, 1.0], [-0.01, 1.0], [-0.03, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-28620.96 -28620.96 -28620.96] [-133638.07 -133638.07 -133638.07] [-333096.58 -333096.58 -333096.58]	[-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10]
CLOopt3.9	[-0.008, 1.0], [-0.009, 1.0], [-0.01, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-110992.28 -110992.28 -110992.28] [-122407.44 -122407.44 -122407.44] [-133638.07 -133638.07 -133638.07]	[-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10]
CLOopt3.10	[-0.004, 1.0], [-0.005, 1.0], [-0.006, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-64041.49 -64041.49 -64041.49] [-76007.34 -76007.34 -76007.34] [-87554.61 -87554.61 -87554.61]	[-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10] [-8006.10 -8006.10 -8006.10]
CLOopt3.11	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0.001, 1.0], [-0.01, 1.0], [-0.03, 1.0]	[-12434.56 -12434.56 -12434.56] [-12434.56 -12434.56 -12434.56] [-12434.56 -12434.56 -12434.56]	[-19136.13 -90545.73 -229558.13] [-19136.13 -90545.73 -229558.13] [-19136.13 -90545.73 -229558.13]
CLOopt3.12	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0.008, 1.0], [-0.009, 1.0], [-0.01, 1.0]	[-12434.56 -12434.56 -12434.56] [-12434.56 -12434.56 -12434.56] [-12434.56 -12434.56 -12434.56]	[-74868.06 -82573.12 -90545.73] [-74868.06 -82573.12 -90545.73] [-74868.06 -82573.12 -90545.73]
CLOopt3.13	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0.004, 1.0], [-0.005, 1.0], [-0.006, 1.0]	[-12434.56 -12434.56 -12434.56] [-12434.56 -12434.56 -12434.56] [-12434.56 -12434.56 -12434.56]	[-43008.00 -51122.83 -58933.20] [-43008.00 -51122.83 -58933.20] [-43008.00 -51122.83 -58933.20]

Table B.9.: Simulation results of the third scenario for the optimized cost function center lane offset

B.4. Simulation results for the optimized cost function Euclidean distance

ID	u_veh1	u_veh2	J_veh1	J_veh2
EDopt1.1	[-0.01, 0.0], [-0.03, 0.0], [-0.05, 0.0]	[-0.000, 0.0], [0.0, 0.0], [0.000, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
EDopt1.2	[-0.03, 0.0], [-0.05, 0.0], [-0.07, 0.0]	[-0.000, 0.0], [0.0, 0.0], [0.000, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
EDopt1.3	[-0.01, 1.0], [-0.03, 1.0], [-0.05, 1.0]	[-0.000, 0.0], [0.0, 0.0], [0.000, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
EDopt1.4	[-0.03, 1.0], [-0.05, 1.0], [-0.07, 1.0]	[-0.000, 0.0], [0.0, 0.0], [0.000, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
EDopt1.5	[-0.005, 0.0], [-0.007, 0.0], [-0.009, 0.0]	[-0.000, 1.0], [0.0, 1.0], [0.000, 1.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
EDopt1.6	[-0.008, 0.0], [-0.009, 0.0], [-0.01, 0.0]	[-0.000, 1.0], [0.0, 1.0], [0.000, 1.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
EDopt1.7	[-0.02, 1.0], [-0.03, 1.0], [-0.04, 1.0]	[-0.000, 1.0], [0.0, 1.0], [0.000, 1.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
EDopt1.8	[-0.01, 1.0], [-0.02, 1.0], [-0.03, 1.0]	[-0.000, 1.0], [0.0, 1.0], [0.000, 1.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]

Table B.10.: Simulation results of the first scenario for the optimized cost function Euclidean distance

ID	u_veh1	u_veh2	J_veh1	J_veh2
EDopt2.1	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
EDopt2.2	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[5541.12 5541.12 5541.12] [5541.12 5541.12 5541.12] [5541.12 5541.12 5541.12]	[3654.02 3654.02 3654.02] [3654.02 3654.02 3654.02] [3654.02 3654.02 3654.02]
EDopt2.3	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
EDopt2.4	[0.003, 1.0], [0.004, 1.0], [0.005, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
EDopt2.5	[0.001, 1.0], [0.002, 1.0], [0.003, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[5328.82 5328.82 5328.82] [5569.75 5569.75 5569.75] [-0. -0. -0.]	[3528.33 3528.33 3528.33] [3687.80 3687.80 3687.80] [-0. -0. -0.]

Table B.11.: Simulation results of the second scenario for the optimized cost function Euclidean distance

ID	u_veh1	u_veh2	J_veh1	J_veh2
EDopt3.1	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
EDopt3.2	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
EDopt3.3	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
EDopt3.4	[0.008, 0.0], [0.009, 0.0], [0.01, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
EDopt3.5	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
EDopt3.6	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
EDopt3.7	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.008, 0.0], [0.009, 0.0], [0.01, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]

Table B.12.: Simulation results of the third scenario for the optimized cost function Euclidean distance

B.5. Simulation results for the optimized cost function obstacle distance

ID	u_veh1	u_veh2	J_veh1	J_veh2
OBSopt1.1	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[2043.42 2043.42 2043.42] [2043.42 2043.42 2043.42] [2043.42 2043.42 2043.42]	[3436.55 3436.55 3436.55] [3436.55 3436.55 3436.55] [3436.55 3436.55 3436.55]
OBSopt1.2	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[2366.80 2366.80 2366.80] [2366.80 2366.80 2366.80] [2366.80 2366.80 2366.80]	[3436.55 3436.55 3436.55] [3436.55 3436.55 3436.55] [3436.55 3436.55 3436.55]
OBSopt1.3	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.008, 0.0], [0.009, 0.0], [0.01, 0.0]	[2043.42 2043.42 2043.42] [2043.42 2043.42 2043.42] [2043.42 2043.42 2043.42]	[2165.64 -0. -0.] [2165.64 -0. -0.] [2165.64 -0. -0.]
OBSopt1.4	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.004, 0.0], [0.005, 0.0], [0.006, 0.0]	[2043.42 2043.42 2043.42] [2043.42 2043.42 2043.42] [2043.42 2043.42 2043.42]	[4325.77 5014.23 3497.05] [4325.77 5014.23 3497.05] [4325.77 5014.23 3497.05]
OBSopt1.5	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.008, 1.0], [0.009, 1.0], [0.01, 1.0]	[2043.42 2043.42 2043.42] [2043.42 2043.42 2043.42] [2043.42 2043.42 2043.42]	[2490.57 -0. -0.] [2490.57 -0. -0.] [2490.57 -0. -0.]
OBSopt1.6	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.004, 1.0], [0.005, 1.0], [0.006, 1.0]	[2043.42 2043.42 2043.42] [2043.42 2043.42 2043.42] [2043.42 2043.42 2043.42]	[5361.65 3452.14 4131.31] [5361.65 3452.14 4131.31] [5361.65 3452.14 4131.31]

Table B.13.: Simulation results of the first scenario for the optimized cost function obstacle distance

ID	u_veh1	u_veh2	J_veh1	J_veh2
OBSopt2.1	[0.005, 0.0], [0.007, 0.0], [0.009, 0.0]	[0.015, 0.0], [0.016, 0.0], [0.017, 0.0]	[6120.38 6120.38 6120.38] [4621.04 4621.04 4621.04] [-0. -0. -0.]	[1768.77 1901.34 2034.48] [1768.77 1901.34 2034.48] [1768.77 1901.34 2034.48]
OBSopt2.2	[0.005, 0.0], [0.007, 0.0], [0.009, 0.0]	[0.015, 1.0], [0.016, 1.0], [0.017, 1.0]	[6120.38 6120.38 6120.38] [4621.04 4621.04 4621.04] [-0. -0. -0.]	[4358.95 4633.48 4910.12] [4358.95 4633.48 4910.12] [4358.95 4633.48 4910.12]
OBSopt2.3	[0.005, 1.0], [0.007, 1.0], [0.009, 1.0]	[0.015, 0.0], [0.016, 0.0], [0.017, 0.0]	[4037.83 4037.83 4037.83] [2224.25 2224.25 2224.25] [3052.56 3052.56 3052.56]	[1768.77 1901.34 2034.48] [1768.77 1901.34 2034.48] [1768.77 1901.34 2034.48]
OBSopt2.4	[0.005, 1.0], [0.007, 1.0], [0.009, 1.0]	[0.015, 1.0], [0.016, 1.0], [0.017, 1.0]	[4037.83 4037.83 4037.83] [2224.25 2224.25 2224.25] [3052.56 3052.56 3052.56]	[4358.95 4633.48 4910.12] [4358.95 4633.48 4910.12] [4358.95 4633.48 4910.12]

Table B.14.: Simulation results of the second scenario for the optimized cost function obstacle distance

ID	u_veh1	u_veh2	J_veh1	J_veh2
OBSopt3.1	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[4542.92 4542.92 4542.92] [4542.92 4542.92 4542.92] [4542.92 4542.92 4542.92]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
OBSopt3.2	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[4542.92 4542.92 4542.92] [4542.92 4542.92 4542.92] [4542.92 4542.92 4542.92]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
OBSopt3.3	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.07, 0.0], [0.08, 0.0], [0.09, 0.0]	[4542.92 4542.92 4542.92] [4542.92 4542.92 4542.92] [4542.92 4542.92 4542.92]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
OBSopt3.4	[0.005, 1.0], [0.007, 1.0], [0.009, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[4037.83 4037.83 4037.83] [2224.25 2224.25 2224.25] [3052.56 3052.56 3052.56]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
OBSopt3.5	[0.003, 1.0], [0.004, 1.0], [0.005, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[5912.57 5912.57 5912.57] [3415.14 3415.14 3415.14] [4037.83 4037.83 4037.83]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
OBSopt3.6	[0.005, 0.0], [0.007, 0.0], [0.009, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[6120.38 6120.38 6120.38] [4621.04 4621.04 4621.04] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
OBSopt3.7	[0.003, 0.0], [0.004, 0.0], [0.005, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[4831.67 4831.67 4831.67] [5414.20 5414.20 5414.20] [6120.38 6120.38 6120.38]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]

Table B.15.: Simulation results of the third scenario for the optimized cost function obstacle distance

B.6. Simulation results for the optimized cost function quadratic velocity

ID	u_veh1	u_veh2	J_veh1	J_veh2
VQopt1.1	[0.0, 0.0], [0.0, 0.0], [0.00, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-9.80 -9.80 -9.80] [-9.80 -9.80 -9.80] [-9.80 -9.80 -9.80]
VQopt1.2	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-9.80 -9.80 -9.80] [-9.80 -9.80 -9.80] [-9.80 -9.80 -9.80]
VQopt1.3	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-9.80 -9.80 -9.80] [-9.80 -9.80 -9.80] [-9.80 -9.80 -9.80]
VQopt1.4	[0.07, 0.0], [0.08, 0.0], [0.09, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-9.80 -9.80 -9.80] [-9.80 -9.80 -9.80] [-9.80 -9.80 -9.80]
VQopt1.5	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-9.80 -9.80 -9.80] [-9.80 -9.80 -9.80] [-9.80 -9.80 -9.80]
VQopt1.6	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-9.80 -9.80 -9.80] [-9.80 -9.80 -9.80] [-9.80 -9.80 -9.80]
VQopt1.7	[0.07, 0.0], [0.08, 0.0], [0.09, 0.0]	[0.07, 0.0], [0.08, 0.0], [0.09, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-9.80 -9.80 -9.80] [-9.80 -9.80 -9.80] [-9.80 -9.80 -9.80]
VQopt1.8	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-9.80 -9.80 -9.80] [-9.80 -9.80 -9.80] [-9.80 -9.80 -9.80]
VQopt1.9	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[-28.71 -28.71 -28.71] [-0.97 -0.97 -0.97] [-0. -0. -0.]	[-2.04 -3.90 -9.80] [-2.04 -3.90 -9.80] [-2.04 -3.90 -9.80]
VQopt1.10	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[-94.56 -94.56 -94.56] [-7.18 -7.18 -7.18] [-0. -0. -0.]	[-2.10 -3.91 -12.96] [-2.10 -3.91 -12.96] [-2.10 -3.91 -12.96]
VQopt1.11	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-98.02 -98.02 -98.02] [-98.02 -98.02 -98.02] [-98.02 -98.02 -98.02]	[-0.47 -0.47 -0.47] [-0.47 -0.47 -0.47] [-0.47 -0.47 -0.47]
VQopt1.12	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0.47 -0.47 -0.47] [-0.47 -0.47 -0.47] [-0.47 -0.47 -0.47]	[-64.35 -64.35 -64.35] [-64.35 -64.35 -64.35] [-64.35 -64.35 -64.35]

Table B.16.: Simulation results of the first scenario for the optimized cost function quadratic velocity

ID	u_veh1	u_veh2	J_veh1	J_veh2
VQopt2.1	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-9.80 -9.80 -9.80] [-9.80 -9.80 -9.80] [-9.80 -9.80 -9.80]
VQopt2.2	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[-28.71 -28.71 -28.71] [-0.97 -0.97 -0.97] [-0. -0. -0.]	[-2.04 -3.90 -9.80] [-2.04 -3.90 -9.80] [-2.04 -3.90 -9.80]
VQopt2.3	[0.0, 1.0], [0.0, 0.5], [0.00, 0.0]	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[-94.56 -94.56 -94.56] [-7.18 -7.18 -7.18] [-0. -0. -0.]	[-2.10 -3.91 -12.96] [-2.10 -3.91 -12.96] [-2.10 -3.91 -12.96]
VQopt2.4	[0.0, 0.0], [0.0, 0.0], [0.00, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-98.02 -98.02 -98.02] [-98.02 -98.02 -98.02] [-98.02 -98.02 -98.02]	[-0.47 -0.47 -0.47] [-0.47 -0.47 -0.47] [-0.47 -0.47 -0.47]
VQopt2.5	[0.0, 0.0], [0.0, 0.0], [0.00, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0.47 -0.47 -0.47] [-0.47 -0.47 -0.47] [-0.47 -0.47 -0.47]	[-64.35 -64.35 -64.35] [-64.35 -64.35 -64.35] [-64.35 -64.35 -64.35]

Table B.17.: Simulation results of the second scenario for the optimized cost function quadratic velocity

ID	u_veh1	u_veh2	J_veh1	J_veh2
VQopt3.1	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-9.80 -9.80 -9.80] [-9.80 -9.80 -9.80] [-9.80 -9.80 -9.80]
VQopt3.2	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[-28.71 -28.71 -28.71] [-0.97 -0.97 -0.97] [-0. -0. -0.]	[-2.04 -3.90 -9.80] [-2.04 -3.90 -9.80] [-2.04 -3.90 -9.80]
VQopt3.3	[0.0, 1.0], [0.0, 0.5], [0.00, 0.0]	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[-94.56 -94.56 -94.56] [-7.18 -7.18 -7.18] [-0. -0. -0.]	[-2.10 -3.91 -12.96] [-2.10 -3.91 -12.96] [-2.10 -3.91 -12.96]
VQopt3.4	[0.0, 0.0], [0.0, 0.0], [0.00, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-98.02 -98.02 -98.02] [-98.02 -98.02 -98.02] [-98.02 -98.02 -98.02]	[-0.47 -0.47 -0.47] [-0.47 -0.47 -0.47] [-0.47 -0.47 -0.47]
VQopt3.5	[0.0, 0.0], [0.0, 0.0], [0.00, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0.47 -0.47 -0.47] [-0.47 -0.47 -0.47] [-0.47 -0.47 -0.47]	[-64.35 -64.35 -64.35] [-64.35 -64.35 -64.35] [-64.35 -64.35 -64.35]

Table B.18.: Simulation results of the third scenario for the optimized cost function quadratic velocity

B.7. Simulation results for the optimized cost function steering angle

ID	u_veh1	u_veh2	J_veh1	J_veh2
STopt1.1	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-28.24 -28.24 -28.24] [-28.24 -28.24 -28.24] [-28.24 -28.24 -28.24]	[-22.93 -22.93 -22.93] [-22.93 -22.93 -22.93] [-22.93 -22.93 -22.93]
STopt1.2	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-31.49 -31.49 -31.49] [-31.49 -31.49 -31.49] [-31.49 -31.49 -31.49]	[-25.81 -25.81 -25.81] [-25.81 -25.81 -25.81] [-25.81 -25.81 -25.81]
STopt1.3	[-0.01, 0.0], [-0.03, 0.0], [-0.05, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-332.64 -332.64 -332.64] [-1050.02 -1050.02 -1050.02] [-1767.24 -1767.24 -1767.24]	[-22.93 -22.93 -22.93] [-22.93 -22.93 -22.93] [-22.93 -22.93 -22.93]
STopt1.4	[-0.001, 0.0], [-0.005, 0.0], [-0.009, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-18.36 -18.36 -18.36] [-154.16 -154.16 -154.16] [-296.91 -296.91 -296.91]	[-22.93 -22.93 -22.93] [-22.93 -22.93 -22.93] [-22.93 -22.93 -22.93]
STopt1.5	[-0.01, 1.0], [-0.03, 1.0], [-0.05, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-384.14 -384.14 -384.14] [-1210.60 -1210.60 -1210.60] [-2037.31 -2037.31 -2037.31]	[-22.93 -22.93 -22.93] [-22.93 -22.93 -22.93] [-22.93 -22.93 -22.93]
STopt1.6	[-0.001, 1.0], [-0.005, 1.0], [-0.009, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-20.91 -20.91 -20.91] [-178.33 -178.33 -178.33] [-342.95 -342.95 -342.95]	[-22.93 -22.93 -22.93] [-22.93 -22.93 -22.93] [-22.93 -22.93 -22.93]
STopt1.7	[-0.01, 0.0], [-0.03, 0.0], [-0.05, 0.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-332.64 -332.64 -332.64] [-1050.02 -1050.02 -1050.02] [-1767.24 -1767.24 -1767.24]	[-25.81 -25.81 -25.81] [-25.81 -25.81 -25.81] [-25.81 -25.81 -25.81]
STopt1.8	[-0.001, 0.0], [-0.005, 0.0], [-0.009, 0.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-18.36 -18.36 -18.36] [-154.16 -154.16 -154.16] [-296.91 -296.91 -296.91]	[-25.81 -25.81 -25.81] [-25.81 -25.81 -25.81] [-25.81 -25.81 -25.81]
STopt1.9	[-0.01, 1.0], [-0.03, 1.0], [-0.05, 1.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-384.14 -384.14 -384.14] [-1210.60 -1210.60 -1210.60] [-2037.31 -2037.31 -2037.31]	[-25.81 -25.81 -25.81] [-25.81 -25.81 -25.81] [-25.81 -25.81 -25.81]
STopt1.10	[-0.001, 1.0], [-0.005, 1.0], [-0.009, 1.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-20.91 -20.91 -20.91] [-178.33 -178.33 -178.33] [-342.95 -342.95 -342.95]	[-25.81 -25.81 -25.81] [-25.81 -25.81 -25.81] [-25.81 -25.81 -25.81]

Table B.19.: Simulation results of the first scenario for the optimized cost function steering angle

ID	u_veh1	u_veh2	J_veh1	J_veh2
STopt2.1	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-28.24 -28.24 -28.24] [-28.24 -28.24 -28.24] [-28.24 -28.24 -28.24]	[-22.93 -22.93 -22.93] [-22.93 -22.93 -22.93] [-22.93 -22.93 -22.93]
STopt2.2	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-31.49 -31.49 -31.49] [-31.49 -31.49 -31.49] [-31.49 -31.49 -31.49]	[-25.81 -25.81 -25.81] [-25.81 -25.81 -25.81] [-25.81 -25.81 -25.81]
STopt2.3	[0.003, 0.0], [0.004, 0.0], [0.005, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-135.49 -135.49 -135.49] [-171.37 -171.37 -171.37] [-207.25 -207.25 -207.25]	[-22.93 -22.93 -22.93] [-22.93 -22.93 -22.93] [-22.93 -22.93 -22.93]
STopt2.4	[0.001, 0.0], [0.002, 0.0], [0.003, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-63.73 -63.73 -63.73] [-99.61 -99.61 -99.61] [-135.49 -135.49 -135.49]	[-22.93 -22.93 -22.93] [-22.93 -22.93 -22.93] [-22.93 -22.93 -22.93]
STopt2.5	[0.003, 1.0], [0.004, 1.0], [0.005, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-155.09 -155.09 -155.09] [-196.44 -196.44 -196.44] [-237.79 -237.79 -237.79]	[-22.93 -22.93 -22.93] [-22.93 -22.93 -22.93] [-22.93 -22.93 -22.93]
STopt2.6	[0.001, 1.0], [0.002, 1.0], [0.003, 1.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-72.39 -72.39 -72.39] [-113.74 -113.74 -113.74] [-155.09 -155.09 -155.09]	[-22.93 -22.93 -22.93] [-22.93 -22.93 -22.93] [-22.93 -22.93 -22.93]

Table B.20.: Simulation results of the second scenario for the optimized cost function steering angle

ID	u_veh1	u_veh2	J_veh1	J_veh2
STopt3.1	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-1463.37 -1463.37 -1463.37] [-1822.33 -1822.33 -1822.33] [-2181.98 -2181.98 -2181.98]	[-22.93 -22.93 -22.93] [-22.93 -22.93 -22.93] [-22.93 -22.93 -22.93]
STopt3.2	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-386.66 -386.66 -386.66] [-745.46 -745.46 -745.46] [-1104.26 -1104.26 -1104.26]	[-22.93 -22.93 -22.93] [-22.93 -22.93 -22.93] [-22.93 -22.93 -22.93]
STopt3.3	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[-28.24 -28.24 -28.24] [-28.24 -28.24 -28.24] [-28.24 -28.24 -28.24]	[-1185.09 -1475.90 -1766.72] [-1185.09 -1475.90 -1766.72] [-1185.09 -1475.90 -1766.72]
STopt3.4	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[-28.24 -28.24 -28.24] [-28.24 -28.24 -28.24] [-28.24 -28.24 -28.24]	[-313.13 -603.76 -894.39] [-313.13 -603.76 -894.39] [-313.13 -603.76 -894.39]
STopt3.5	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[-386.66 -386.66 -386.66] [-745.46 -745.46 -745.46] [-1104.26 -1104.26 -1104.26]	[-313.13 -603.76 -894.39] [-313.13 -603.76 -894.39] [-313.13 -603.76 -894.39]
STopt3.6	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[-1463.37 -1463.37 -1463.37] [-1822.33 -1822.33 -1822.33] [-2181.98 -2181.98 -2181.98]	[-1185.09 -1475.90 -1766.72] [-1185.09 -1475.90 -1766.72] [-1185.09 -1475.90 -1766.72]
STopt3.7	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[-386.66 -386.66 -386.66] [-745.46 -745.46 -745.46] [-1104.26 -1104.26 -1104.26]	[-1185.09 -1475.90 -1766.72] [-1185.09 -1475.90 -1766.72] [-1185.09 -1475.90 -1766.72]
STopt3.8	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[-1463.37 -1463.37 -1463.37] [-1822.33 -1822.33 -1822.33] [-2181.98 -2181.98 -2181.98]	[-313.13 -603.76 -894.39] [-313.13 -603.76 -894.39] [-313.13 -603.76 -894.39]
STopt3.9	[0.01, 1.0], [0.02, 1.0], [0.03, 1.0]	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[-444.53 -444.53 -444.53] [-858.03 -858.03 -858.03] [-1271.62 -1271.62 -1271.62]	[-313.13 -603.76 -894.39] [-313.13 -603.76 -894.39] [-313.13 -603.76 -894.39]
STopt3.10	[0.04, 1.0], [0.05, 1.0], [0.06, 1.0]	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[-1685.58 -1685.58 -1685.58] [-2099.16 -2099.16 -2099.16] [-2513.21 -2513.21 -2513.21]	[-1185.09 -1475.90 -1766.72] [-1185.09 -1475.90 -1766.72] [-1185.09 -1475.90 -1766.72]
STopt3.11	[0.01, 1.0], [0.02, 1.0], [0.03, 1.0]	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[-444.53 -444.53 -444.53] [-858.03 -858.03 -858.03] [-1271.62 -1271.62 -1271.62]	[-1185.09 -1475.90 -1766.72] [-1185.09 -1475.90 -1766.72] [-1185.09 -1475.90 -1766.72]
STopt3.12	[0.04, 1.0], [0.05, 1.0], [0.06, 1.0]	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[-1685.58 -1685.58 -1685.58] [-2099.16 -2099.16 -2099.16] [-2513.21 -2513.21 -2513.21]	[-313.13 -603.76 -894.39] [-313.13 -603.76 -894.39] [-313.13 -603.76 -894.39]
STopt3.13	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[0.01, 1.0], [0.02, 1.0], [0.03, 1.0]	[-386.66 -386.66 -386.66] [-745.46 -745.46 -745.46] [-1104.26 -1104.26 -1104.26]	[-365.44 -705.51 -1045.66] [-365.44 -705.51 -1045.66] [-365.44 -705.51 -1045.66]
STopt3.14	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[0.04, 1.0], [0.05, 1.0], [0.06, 1.0]	[-1463.37 -1463.37 -1463.37] [-1822.33 -1822.33 -1822.33] [-2181.98 -2181.98 -2181.98]	[-1385.81 -1726.34 -2066.94] [-1385.81 -1726.34 -2066.94] [-1385.81 -1726.34 -2066.94]
STopt3.15	[0.01, 0.0], [0.02, 0.0], [0.03, 0.0]	[0.04, 1.0], [0.05, 1.0], [0.06, 1.0]	[-386.66 -386.66 -386.66] [-745.46 -745.46 -745.46] [-1104.26 -1104.26 -1104.26]	[-1385.81 -1726.34 -2066.94] [-1385.81 -1726.34 -2066.94] [-1385.81 -1726.34 -2066.94]
STopt3.16	[0.04, 0.0], [0.05, 0.0], [0.06, 0.0]	[0.01, 1.0], [0.02, 1.0], [0.03, 1.0]	[-1463.37 -1463.37 -1463.37] [-1822.33 -1822.33 -1822.33] [-2181.98 -2181.98 -2181.98]	[-365.44 -705.51 -1045.66] [-365.44 -705.51 -1045.66] [-365.44 -705.51 -1045.66]
STopt3.17	[0.01, 1.0], [0.02, 1.0], [0.03, 1.0]	[0.01, 1.0], [0.02, 1.0], [0.03, 1.0]	[-444.53 -444.53 -444.53] [-858.03 -858.03 -858.03] [-1271.62 -1271.62 -1271.62]	[-365.44 -705.51 -1045.66] [-365.44 -705.51 -1045.66] [-365.44 -705.51 -1045.66]
STopt3.18	[0.04, 1.0], [0.05, 1.0], [0.06, 1.0]	[0.04, 1.0], [0.05, 1.0], [0.06, 1.0]	[-444.53 -444.53 -444.53] [-858.03 -858.03 -858.03] [-1271.62 -1271.62 -1271.62]	[-1385.81 -1726.34 -2066.94] [-1385.81 -1726.34 -2066.94] [-1385.81 -1726.34 -2066.94]
STopt3.19	[0.01, 1.0], [0.02, 1.0], [0.03, 1.0]	[0.04, 1.0], [0.05, 1.0], [0.06, 1.0]	[-444.53 -444.53 -444.53] [-858.03 -858.03 -858.03] [-1271.62 -1271.62 -1271.62]	[-1385.81 -1726.34 -2066.94] [-1385.81 -1726.34 -2066.94] [-1385.81 -1726.34 -2066.94]

Table B.21.: Simulation results of the third scenario for the optimized cost function steering angle

B.8. Simulation results for the optimized cost function acceleration work

ACopt1.1	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[0.0, 0.0], [0.0, 0.0], [0.0, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
ACopt1.2	[-0.004, 0.0], [-0.005.0, 0.0], [-0.006, 0.0]	[-0.004, 0.0], [-0.005.0, 0.0], [-0.006, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
ACopt1.3	[-0.01, 0.0], [-0.02, 0.0], [-0.03, 0.0]	[-0.01, 0.0], [-0.02, 0.0], [-0.03, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
ACopt1.4	[-0.04, 0.0], [-0.05, 0.0], [-0.06, 0.0]	[-0.04, 0.0], [-0.05, 0.0], [-0.06, 0.0]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]	[-0. -0. -0.] [-0. -0. -0.] [-0. -0. -0.]
ACopt1.5	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-0.03 -0.03 -0.03] [-0.03 -0.03 -0.03] [-0.03 -0.03 -0.03]	[-0.03 -0.03 -0.03] [-0.03 -0.03 -0.03] [-0.03 -0.03 -0.03]
ACopt1.6	[0.0, 0.5], [0.0, 0.5], [0.0, 0.5]	[0.0, 0.5], [0.0, 0.5], [0.0, 0.5]	[-0.009 -0.009 -0.009] [-0.009 -0.009 -0.009] [-0.009 -0.009 -0.009]	[-0.009 -0.009 -0.009] [-0.009 -0.009 -0.009] [-0.009 -0.009 -0.009]
ACopt1.7	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[-0.03 -0.03 -0.03] [-0.009 -0.009 -0.009] [-0. -0. -0.]	[-0.03 -0.03 -0.03] [-0.009 -0.009 -0.009] [-0. -0. -0.]
ACopt1.8	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[-0.09 -0.09 -0.09] [-0.02 -0.02 -0.02] [-0. -0. -0.]	[-0.03 -0.03 -0.03] [-0.009 -0.009 -0.009] [-0. -0. -0.]

Table B.22.: Simulation results of the first scenario for the optimized cost function acceleration work

ID	u_veh1	u_veh2	J_veh1	J_veh2
ACopt2.1	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-0.03 -0.03 -0.03] [-0.03 -0.03 -0.03] [-0.03 -0.03 -0.03]	[-0.03 -0.03 -0.03] [-0.03 -0.03 -0.03] [-0.03 -0.03 -0.03]
ACopt2.2	[0.0, 0.5], [0.0, 0.5], [0.0, 0.5]	[0.0, 0.5], [0.0, 0.5], [0.0, 0.5]	[-0.009 -0.009 -0.009] [-0.009 -0.009 -0.009] [-0.009 -0.009 -0.009]	[-0.009 -0.009 -0.009] [-0.009 -0.009 -0.009] [-0.009 -0.009 -0.009]
ACopt2.3	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[-0.03 -0.03 -0.03] [-0.009 -0.009 -0.009] [-0. -0. -0.]	[-0.03 -0.03 -0.03] [-0.009 -0.009 -0.009] [-0. -0. -0.]
ACopt2.4	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[-0.09 -0.09 -0.09] [-0.02 -0.02 -0.02] [-0. -0. -0.]	[-0.03 -0.03 -0.03] [-0.009 -0.009 -0.009] [-0. -0. -0.]

Table B.23.: Simulation results of the second scenario for the optimized cost function acceleration work

ID	u_veh1	u_veh2	J_veh1	J_veh2
ACopt3.1	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[0.0, 1.0], [0.0, 1.0], [0.0, 1.0]	[-0.03 -0.03 -0.03] [-0.03 -0.03 -0.03] [-0.03 -0.03 -0.03]	[-0.03 -0.03 -0.03] [-0.03 -0.03 -0.03] [-0.03 -0.03 -0.03]
ACopt3.2	[0.0, 0.5], [0.0, 0.5], [0.0, 0.5]	[0.0, 0.5], [0.0, 0.5], [0.0, 0.5]	[-0.009 -0.009 -0.009] [-0.009 -0.009 -0.009] [-0.009 -0.009 -0.009]	[-0.009 -0.009 -0.009] [-0.009 -0.009 -0.009] [-0.009 -0.009 -0.009]
ACopt3.3	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[-0.03 -0.03 -0.03] [-0.009 -0.009 -0.009] [-0. -0. -0.]	[-0.03 -0.03 -0.03] [-0.009 -0.009 -0.009] [-0. -0. -0.]
ACopt3.4	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[0.0, 1.0], [0.0, 0.5], [0.0, 0.0]	[-0.09 -0.09 -0.09] [-0.02 -0.02 -0.02] [-0. -0. -0.]	[-0.03 -0.03 -0.03] [-0.009 -0.009 -0.009] [-0. -0. -0.]

Table B.24.: Simulation results of the third scenario for the optimized cost function acceleration work