Collision-free first order pedestrian model

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Motivations

Inertial force-based (Newtonian) pedestrian models described many features of crowd dynamics, yet inertia potentially induces

- Local oscillations (cf. harmonic oscillator / 2nd order equation)
- Collision (overlapping) and uncontrolled motion backward
- Restricted parameter values or adding of parameters and speed difference terms to get realistic dynamics / Difficult calibration
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→ Development of collision-free first order models

Overview

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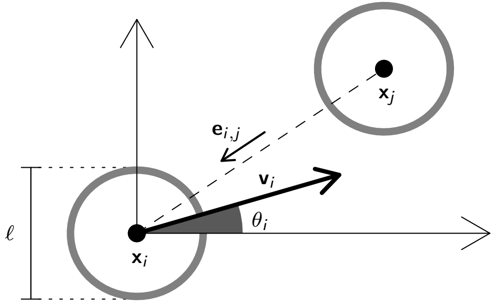
1. Existing first order pedestrian models
2. Collision-free property
3. OV model
4. Model for the direction
5. Simulation results
6. Conclusions and working perspectives

Notations



$\mathbf{x}_i$	Position of pedestrian i	ℓ	Pedestrian size
$\mathbf{v}_i$	Speed of pedestrian i	$\mathbf{e}_{i,j}$	Unit vector from $\mathbf{x}_j$ to $\mathbf{x}_i$
θ_i	Direction of pedestrian i	$\mathbf{e}_i = (\cos\theta_i, \sin\theta_i)$	$s_{i,j} = \|\|\mathbf{x}_i - \mathbf{x}_j\|\|$

Existing first order pedestrian models

First order models : Velocity function

$$\dot{\mathbf{x}}_i = \mathbf{v}(\mathbf{x}_i, \mathbf{x}_j, \ldots) \qquad \text{or} \qquad \dot{\mathbf{x}}_i = V(\mathbf{x}_i, \mathbf{x}_j, \ldots) \times \mathbf{e}_i(\mathbf{x}_i, \mathbf{x}_j, \ldots)$$

with V the speed function and $\mathbf{e}_i$ the direction

[7] F. Dietrich and G. Köster *Phys. Rev. E* 89:062801 (2014)

[8] J. Ondřej et al. In ACM Trans. Graph. 29:123 (2010)

[9] Fiorini and Shiller *Int. J. Robot. Res.* 17(7):760 (1998)

[10] B. Maury and J. Venel *ESAIM-Math. Model. Num.* 45:145 (2011)

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- *Gradient navigation model* with additive neighbour repulsions[7]
- *Synthetic-vision-based model* based on time-to-interaction and bearing angle [8]
- *Velocity obstacle approach* borrowed from robotic[9]
- Maury and Venel mathematical framework and evacuation model[10]

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Gradient navigation model[11]

First order model with relaxed speed

$$\begin{cases} \dot{\mathbf{x}}_i &= \omega_i \, \mathbf{e}_i(\mathbf{x}_i, \mathbf{x}_j, \ldots) \\ \dot{\omega}_i &= \frac{1}{\tau}(v_0 ||\mathbf{e}_i(\mathbf{x}_i, \mathbf{x}_j, \ldots)|| - \omega_i) \end{cases}$$

with v_0 the (scalar) desired speed, τ the reaction time and

$$\mathbf{e}_i = g\Big(g\left(\mathbf{e}_0\right) + g\big(-\textstyle\sum_j r(s_{i,j}) \, w_{i,j} \, \mathbf{e}_{i,j}\big)\Big)$$

with $\mathbf{e}_0$ the desired direction, $w_{i,j}$ a vision angle weight, r the repulsion, and g scaling function such that $||g(\mathbf{x})|| \to 0$ and $||g(\mathbf{x})|| \to 1$ as $||\mathbf{x}|| \to 0$ and $||\mathbf{x}|| \to \infty$

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$\rightarrow$ Additive repulsions with neighbours like force-based models

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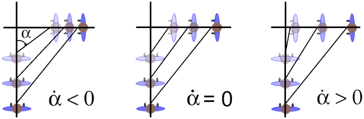
Synthetic-vision-based steering model[12]



Source: See footnote 12

'Vision based' model depending on :

- Bearing-angle α and its derivative
- Time-to-interaction TTI (time-to-collision)

Velocity function $V(TTI_i) = v_0\big(1 - \exp\big(-0.5\min_j TTI_{i,j}^2\big)\big)$ and discrete direction model (turning right, turning left, or going to desired cap) depending on $(\alpha, \dot{\alpha}, TTI)$

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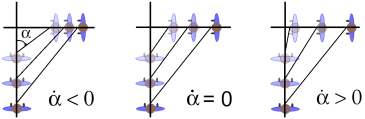

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$\rightarrow$ Description of complex collective structures avoiding gridlocks

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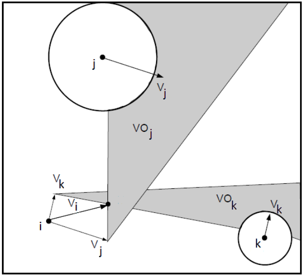
Velocity obstacle model[14]

Approach based on velocity obstacle sets *VO* leading to collisions if the obstacle speeds remain constant

For a desired speed $\mathbf{v}_0$, the velocity is

$$\dot{\mathbf{x}}_i \;=\; \underset{\mathbf{v}\notin\cup_j VO_j}{\arg\min}\; ||\mathbf{v}-\mathbf{v}_0||$$



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[13]V.D. Berg et al. In IEEE International Conference on Robotics and Automation 1928 (2008)
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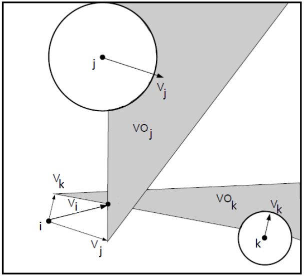

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→ Collision-free dynamics in discrete time if step smaller than horizon time

→ Reciprocal velocity obstacle model to avoid oscillation[13]

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Condition for collision-free dynamics[16]

If pedestrians are considered as disc with diameter ℓ, the set of collision-free configurations is

$$Q_i = \left\{ \mathbf{x}_i \in \mathbb{R}^2,\ s_{i,j} \geq \ell\ \forall j \right\}, \quad \forall i$$

The set of collision-free velocities $C_{\mathbf{x}_i}$ is

$$C_{\mathbf{x}_i} = \left\{ \mathbf{v} \in \mathbb{R}^4,\ s_{i,j} = \ell\ \Rightarrow\ \mathbf{e}_{i,j} \cdot (\mathbf{v}_i - \mathbf{v}_j) \geq 0 \right\}$$

- If $\mathbf{x}_i(0) \in Q_i$ then $\mathbf{x}_i$ remains in Q_i for the dynamics in $C_{\mathbf{x}_i}$[15]

[15] Invariant set, see Monneau et al. NoDEA 21(4):491 (2014) for similar approach with second order model
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Evacuation model by projection of desired speed $\dot{\mathbf{x}}_i = \arg\min_{C_{\mathbf{x}_i}} ||\mathbf{v} - \mathbf{v}_0||$

$\rightarrow$ Pedestrians go as fast as possible (evacuation)

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Literature summary

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- First order models are by construction collision-free if $\mathbf{e}_{i,j} \cdot \mathbf{v}_i \geq 0$ and $\mathbf{e}_{j,i} \cdot \mathbf{v}_j \geq 0$ when $s_{i,j} = \ell$
- Additive forms are easily computable
- Models based on neighbour instantaneous speed can give oscillations

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- Additive forms are easily computable
- Models based on neighbour instantaneous speed can give oscillations

$\rightarrow$ Development of a minimal collision-free model with additive repulsion based on the spacing distances with the neighbours

Optimal velocity (OV) models

Models based on the relation between the speed and the spacing (OV function)

$\rightarrow$ Initially introduce in traffic flow[17]; Also used for pedestrian modelling[18]

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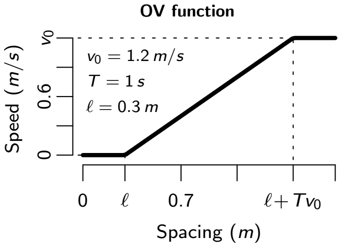
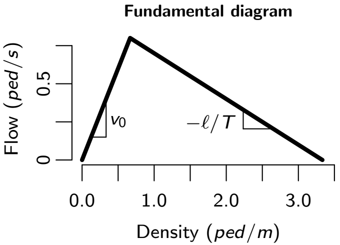
Easy control of fundamental diagram (cf. homogeneous configuration)

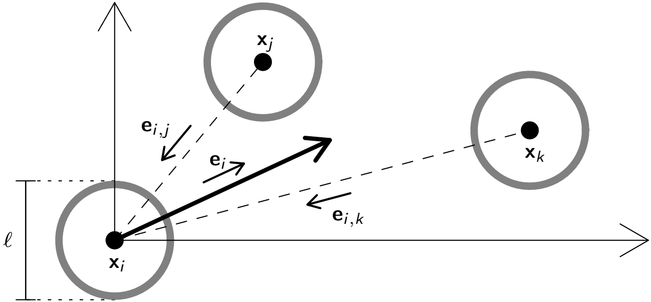




[17] M. Bando et al. *Phys. Rev. E* 51, 1035 (1995)

[18] A. Nakayama et al. *Phys. Rev. E* 71, 036121 (2005)

Minimum distance in front



The pedestrians in front are the set

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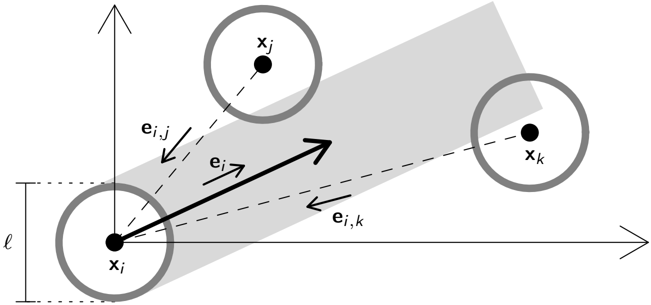

The pedestrians in front are the set $J_i = \{\, j,\ \mathbf{e}_i \cdot \mathbf{e}_{i,j} \le 0 \text{ and } |\mathbf{e}_i^\perp \cdot \mathbf{e}_{i,j}| \le \ell/s_{i,j} \}$

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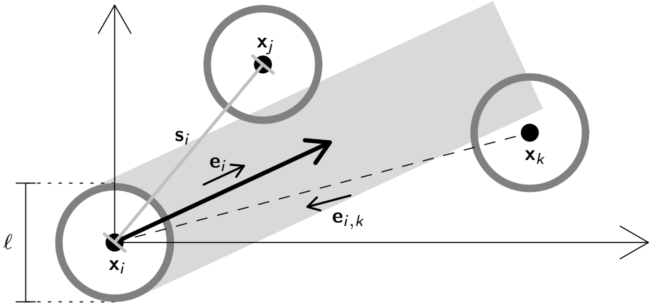

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$\rightarrow$ The minimum distance in front is $s_i = \min_{j \in J_i} s_{i,j}$

Collision-free OV model in two dimensions

Speed and direction models : $\dot{\mathbf{x}}_i = V\big(\mathbf{x}_i, \mathbf{x}_j, \dots\big) \times \mathbf{e}_i(\mathbf{x}_i, \mathbf{x}_j, \dots)$

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- If $\mathcal{V}(.) \geq 0$ and $\mathcal{V}(s) = 0$ for all $s \leq \ell$, the OV model based on minimum distance in front
$$\dot{\mathbf{x}}_i = \mathcal{V}\big(s_i(\mathbf{x}_i, \mathbf{x}_j, \dots)\big) \times \mathbf{e}_i(\mathbf{x}_i, \mathbf{x}_j, \dots)$$
is collision-free for any direction model $\mathbf{e}_i(\cdot)$

Proof For $s_{i,j} = \ell$, if $\mathbf{e}_i \cdot \mathbf{e}_{i,j} \leq 0$ then $j \in J_i$, i.e. $s_i \leq s_{i,j} = \ell$ and $\mathcal{V}(s_i) = 0$, if $\mathbf{e}_i \cdot \mathbf{e}_{i,j} \geq 0$, then $\mathcal{V}(s_i) \geq 0$ since $\mathcal{V}(\cdot) \geq 0$; Therefore $\mathbf{v}_i \cdot \mathbf{e}_{i,j} = \mathcal{V}(s_i) \times \mathbf{e}_i \cdot \mathbf{e}_{i,j} \geq 0$ (similar proof for j)

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$\rightarrow$ Direction model $\mathbf{e}_i(\mathbf{x}_i, \mathbf{x}_j, \dots)$ to define

Direction model

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Exponential additive form (Gradient navigation model) :

$$\mathbf{e}_i(\mathbf{x}_i, \mathbf{x}_j, \dots) = \tfrac{1}{N}\left(\mathbf{e}_0 + \textstyle\sum_j r(s_{i,j})\,\mathbf{e}_{i,j}\right)$$

with N a normalisation constant and $r(s) = a\,\exp\big((\ell - s)/D\big)$ the repulsion

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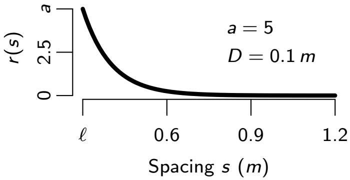
Two parameters :

- Repulsion rate $a > 0$
- Repulsion distance $D > 0$



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Simulation of the model

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5 parameters: **Ped. size** $\ell = 0.3\,m$; **Desired speed** $v_0 = 1.2\,m/s$; **Time gap** $T = 1\,s$
Repulsion rate $a = 5$; **Repulsion distance** $D = 0.1\,m$

Simulation using Euler explicit scheme with time step $dt > 0$

`For each time step` t `in` $dt\,\mathbb{N}$

`For each pedestrian` i

$$\left\| \begin{array}{lcl} \mathbf{e}_i(t) & = & \frac{1}{N}\Big(\mathbf{e}_0(x_i(t),t) + \sum_j r(s_{i,j}(t))\,\mathbf{e}_{i,j}(t)\Big) \\ J_i(t) & = & \big\{\, j,\ \mathbf{e}_i(t)\cdot\mathbf{e}_{i,j}(t) \leq 0 \text{ and } |\mathbf{e}_i^{\perp}(t)\cdot\mathbf{e}_{i,j}(t)| \leq \ell/s_{i,j}(t)\big\} \\ s_i(t) & = & \min_{j\in J_i} s_{i,j}(t) \\ \mathbf{x}_i(t+dt) & = & \mathbf{x}_i(t) + dt \times \mathcal{V}(s_i(t))\,\mathbf{e}_i(t) \end{array}\right.$$

Collision avoidance

Non-overlapping

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Linear flow in bottlenecks

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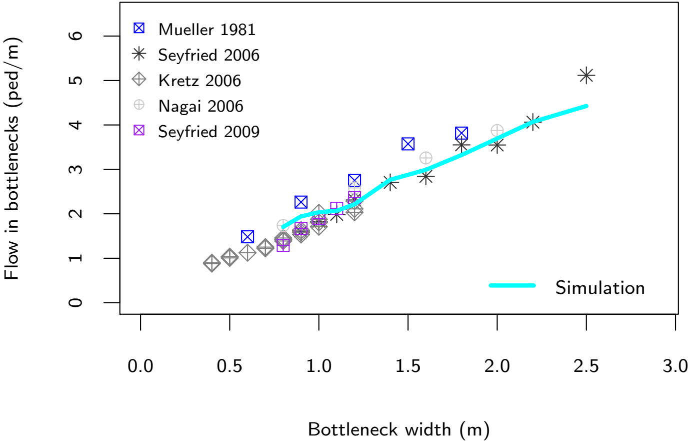
Linear flow in bottlenecks



Lane formation

Up to $4\ ped/m^2$ **for** $\ell = 0.4\ m$ **and** $7\ ped/m^2$ **for** $\ell = 0.3\ m$

Loading...

Freezing by heating effect

Adding of a noise

Loading...

Intermittent bottleneck flows

Loading. . .

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- Collision-free first order pedestrian model in two dimensions
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  Repulsion rate and distance (Direction model)
- Catching of expected properties and self-organized phenomena

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Limitations / Outlooks

- No anticipation (no speed difference) / No reaction time / No vision effect
- No stop-and-go phenomena for congested flow (first order)
- Gridlock for narrow bottlenecks (circle shape)