Misanthrope process for large-scale simulation of pedestrian dynamics

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Forschungszentrum Jülich

♦ Multidisciplinary research center
  - Health
  - Energy
  - Environment
  - Information technology

♦ Approx. 5000 employees

♦ Jülich Supercomputing Centre – Division Civil Safety and Traffic
  - Experimentation and modelling of pedestrian dynamics
  - Fire and evacuation simulation
  - Safety of large-scale events
  - Collaboration with Wuppertal and Cologne Universities
Motivations

♦ Nowadays more than **half of mankind lives in cities**

♦ **Dense crowds** are frequent in train stations, fairs, city centers or during large-scale events (sport, spectacle, concert, demonstration...)

♦ Knowledge of pedestrian dynamics is important for the design and optimization of facilities with respect to **safety or level of service**

♦ **Complex system**: experimentation, data collection, modelling and simulation of pedestrian dynamics are necessary
Misanthrope process

- Borrowed from *Interacting Particle Systems* widely studied in theoretical physics\(^3\) (see also zero-range, exclusion, or mean average processes)

- **Continuous time Markovian jump process** describing evolution of particles in a lattice

- **Unique stationary distribution** (finite set) that can easily be calculated by simulation (Monte Carlo experiments)

- **Misanthrope process**: Each site can contain several particles and the jump rate depends on particle numbers in departure and arrival sites\(^4\)

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\(^3\) T Liggett (1985) *Interacting particle systems* Springer

\(^4\) C Cocozza-Thivent (1985) *Z Wahr Verw Gebiete* 70:509-523
Pedestrian model

Hexagonal lattice with $a > 0$ the face length (area $\alpha = 1.5\sqrt{3}a^2$)
Each hexagon can contain $n \in [0, N]$ pedestrians, $N \geq 1$
Jump rate $b$ to define

$$
\rho_m \propto b(n, n_1) + b(n, n_2) + b(n, n_3) + b(n, n_4) + b(n, n_5) + b(n, n_6)
$$
Model characteristics

- **Discrete space / Continuous time** (also for the simulation)
- **Intrinsically stochastic** (jump times exponentially distributed)
- **Several pedestrians by cell** (size cell sufficiently big)
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- Pedestrians jump individually from one cell to one of the six neighbors
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→ **Mesoscopic approach**: Pedestrians are individually considered but their dynamics are aggregated by cell

→ **Exclusion model for** $N = 1$ (size of the cell = size of a pedestrian)
Jump rate function

♦ **The jump rate** of a pedestrian from a cell with \( n \geq 1 \) pedestrian to cell \( i \) with \( n_i \geq 0 \) pedestrians is

\[
b_i(n, n_i) = \kappa \times J(n, n_i) \times D_i(J(n, n_i))
\]  

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- The flow \( J(n, n_i) \) is the **minimum between the demand** of the considered cell and **the supply** of the destination cell \( i \):

\[
J(n, n_i) = \min\{\Delta(n/\alpha), \Sigma(n_i/\alpha)\}
\]  

(2)
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(2)

♦ The selected direction \( D_i(J(n, n_i)) \) maximizes the weighted flow to the desired direction \( h \):

\[
D_i(J(n, n_i)) = \begin{cases} 
1 & \text{if } f(h - h_i)J(n, n_i) = \max_i f(h - h_i)J(n, n_i) \\
0 & \text{otherwise}
\end{cases}
\]

(3)
Model parameters

Supply $\Sigma(\cdot)$ and demand $\Delta(\cdot)$ functions (fundamental diagram)

Weight $f(\cdot)$ for the desired direction (here $x \mapsto 1 + \cos(x)$)
Simulation of the model

Each cell with at least one pedestrian has an exponential clock

\[ T_0 = t + \mathcal{E}(b) \]  

(4)
Simulation of the model

- Each cell with at least one pedestrian has an **exponential clock**

\[ T_0 = t + \mathcal{E}(b) \]  \hspace{1cm} (4)

- **Event-based simulation in continuous time** by taking successive minimum jump times:

  1. **Step 1.** Select the cell with minimal jump time
  2. **Step 2.** Set time to selected cell jump time / Do the jump
  3. **Step 3.** Update jump times of the cells where jump rate \( b \) changed
  4. **Step 4.** Return to step 1
Simulation of the model

1. Select cell with minimal jump time
2. Jump of a pedestrian
   Global time $t = T_0$
3. Update cell jump time where rate changed

Repeat
Simulation of uni-directional flows

Snapshots in stationary state according to $a$
Simulation of uni-directional flows

Fundamental diagram in stationary state according to $a$

Density ($\text{ped/m}^2$) vs. Flow ($\text{ped/s/m}$)

Density ($\text{ped/m}^2$) vs. Dens std-dev ($\text{ped/m}^2$)
Presence of obstacles
Mean performances in stationary state
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Multi-directional flow model

- $d \in \mathbb{N}^*$ possible desired directions $(h_1, h_2, \ldots, h_d)$

$\rightarrow$ System described by pedestrian numbers by direction $(n^{h_1}, \ldots, n^{h_d})$

Proportion of pedestrians by direction

$$p^h = \frac{n^h}{\sum_h n^h}$$ (5)
Multi-directional flow model

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- Jump rate for the pedestrians with direction $h$ to cell $i$:

  $$b^h_i(n, n_i) = p^h \times b_i(n, n_i)$$  (6)

Proportion $p^h$ of total flow affected to pedestrians with direction $h$

Uni-directional model if only one direction exists ($p^h = 1$)
Counter flows
Random initial condition

\[ \rho = 2.5 \text{ ped} / \text{m}^2 \quad a = 2.5 \text{ m} \]
Multi-directional flow model (2)

- **Total flow bounded** by the proportion by direction to model frictions for pedestrians with different directions

\[ J(n, n_i) \rightarrow J^h(n, n_i) = \min\{ \tilde{p}^h_i Q, J(n, n_i) \} \]  

(7)

with \( \tilde{p}^h = p_0 + (1 - p_0) p^h \) and new parameter \( p_0 \in [0, 1] \)
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- Same model as previous one if \( p_0 = 1 \) (\( J^h = J \) for all \( h \))
- If \( p_0 = 0 \), then \( \tilde{p}^h = p^h \): the jump rates to cells that do not contain any pedestrian with the same direction are nil.
Bounded fundamental diagram

\[ Q = Q_0 \rho_0 \]

\[ \rho^h \]

\[ \rho_0 = 0.2 \]

Flow (ped/s/m) vs Density (ped/m²)
Counter flows
Random initial condition

\[ \rho_0 = 0.2 \quad \rho = 2.5 \text{ ped/m}^2 \quad a = 2.5 \text{ m} \]

\[ J(\text{ped/m/s}) \]

\[ \bar{\sigma}(\text{ped/m}^2) \]

\[ 0 \quad 200 \quad 600 \quad 1000 \]

\[ 0.0 \quad 0.6 \quad 1.2 \]

\[ 0.0 \quad 1.0 \]

Time (s)

Density, \( h = \frac{\pi}{2} \)
Density, \( h = -\frac{\pi}{2} \)
Counter flows
Random initial condition

\[ p_0 = 0.2 \quad \rho = 4 \text{ ped/m}^2 \quad a = 2.5 \text{ m} \]
Counter flows
Random initial condition

\[ p_0 = 0.2 \quad \rho = 4 \text{ ped/m}^2 \quad a = 4 \text{ m} \]
Counter flows
Performances in stationary state

\[ p_0 = 0.2 \quad \rho = 2.5 \text{ ped/m}^2 \quad a = 2.5 \text{ m} \]

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50 experiments per parameter value; Line: mean value; Grey area: Min-max interval

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Description of realistic fundamental diagrams, congestion/rarefaction and lane formation for large cells (i.e. low variability – *Freezing by Heating effect*)
Working perspectives

♦ Comparison to classical microscopic (force-based) and macroscopic (CTM or queuing models) approaches
→ Complexity, realism level, described phenomena
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   → Potential application scales and limits of the model
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♦ Model to understand ⇔ Model to predict
  → Technical and strategic planning motion modelling + other mechanisms
  → Large-scale simulation of pedestrian dynamics