Interinstitutional Research Day on Crowd Management

Forschungszentrum Juelich, Germany – February 18, 2025

Noise-induced stop-and-go dynamics: Modelling and control

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DFG Research Grant *SmartACC* (Gepris 546728715)

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Andreas Schadschneider[3], Oscar Dufour, Alexandre Nicolas, David Rodney[4]

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Outline

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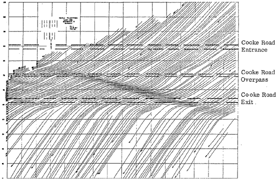
Stop-and-go dynamics in *human-driven* flows

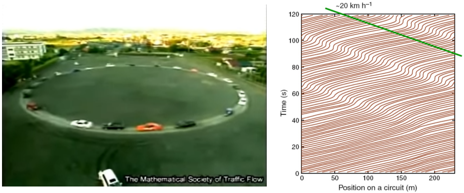
- **Pedestrian, bicycle and car** single-file motions tend to describe **stop-and-go dynamics** for congested density levels

- Succession of **braking** (shock) and **acceleration** (rarefaction) sequences
  $\rightarrow$ *Accordion traffic*
- **Self-organized collective phenomenon**
- Besides its scientific interest, stop-and-go waves have **negative impact on safety, comfort and environment**
- Still **not well understood**, notably for adaptive cruise control (ACC) advanced driver-assistance systems

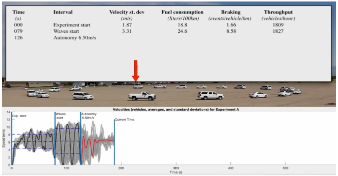
Stop-and-go waves in traffic flow



J Treiterer: Investigation of traffic dynamics
by aerial photogrammetry techniques
EES-278 Final Rpt, 1975



Y Sugiyama et al.: Traffic jams without bottlenecks
New J Phys, 10:033001, 2008*

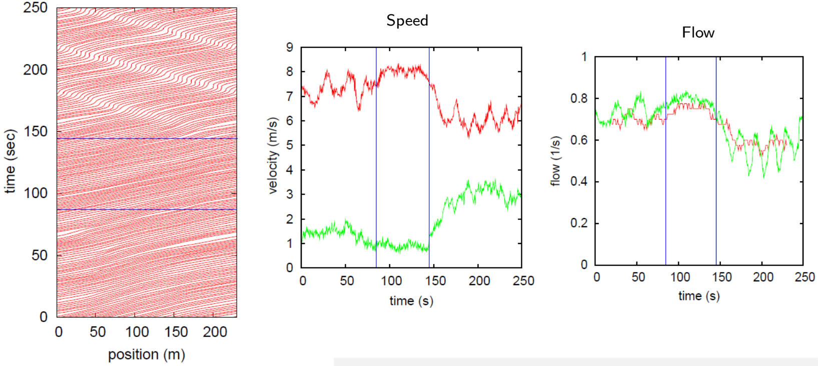


RE Stern et al. Dissipation of stop-and-go
waves via control of autonomous vehicles
Transp Res C-Emerg 89:205, 2018*

`www.trafficforum.org`*

Observation of metastability and phase transition

A Schadschneider et al. *Stochastic Transport in Complex Systems*, Springer, 2010.

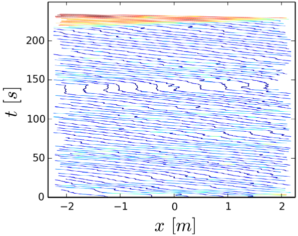


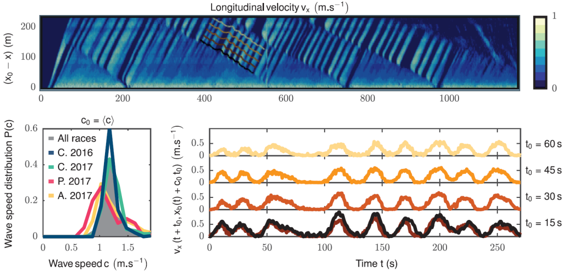
	Speed	Flow
Red curves:	Mean value	Local flow
Green curves:	Standard deviation	$J = V \times \rho$

Stop-and-go in pedestrian dynamics

N Bain & D Bartolo. Dynamic response and hydrodynamics of polarized crowds *Science* 363(6422):46, 2019**

A Portz & A Seyfried: *Pedestrian and Evacuation Dynamics*, pp. 577-586, Springer, 2011

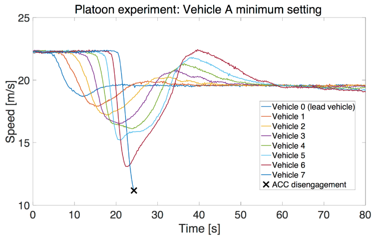




Are commercially implemented adaptive cruise control systems stable?

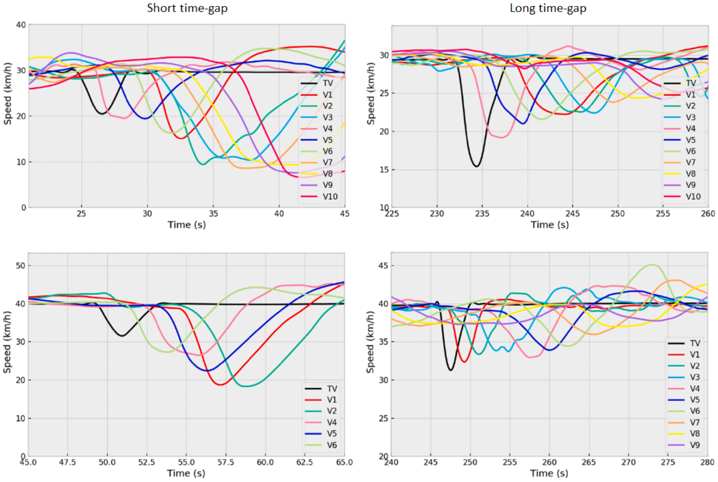
G Gunter et al. *IEEE Trans Intell Transp Sys* 22(11):6992, 2020

- Experimental test with eight 2018 model year ACC equipped vehicles
- Initial disturbance of 10 km/h



Further experiments with ACC systems

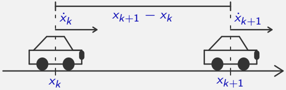
M Makridis et al. OpenACC: An open database of car-following experiments to study the properties of commercial ACC systems *TRC* 125:103047, 2021



Outline

Stop-and-go dynamics in deterministic traffic models

- Stop-and-go by means of **string instability** of the homogeneous configurations



- Examples
  - Delayed 1st order model by Newell (1961) $\qquad \dot{x}_k(t+\tau) = V\big(x_{k+1}(t) - x_k(t)\big)$
  - 2nd order OVM by Bando et al. (1995) $\qquad \ddot{x}_k(t) = \frac{1}{\tau}\big(V(x_{k+1}(t) - x_k(t)) - \dot{x}_k(t)\big)$

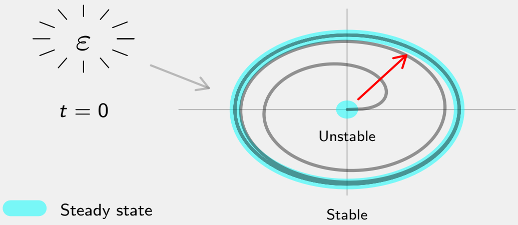
→ **Unstable if inertia τ exceeds critical value:** $\qquad \tau > (2V')^{-1} = T/2$

- Unstable models may have **periodic solutions** (limit-cycle) with stop-and-go waves for **nonlinear models** and **fine tuning of the parameters**
  - Phase transition from uniform equilibrium to oscillating dynamics

Stop-and-go dynamics in deterministic traffic models

Perturbed system: phase transition

Undamped oscillation activated by
the initial perturbation

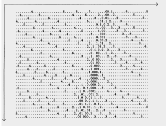


Outline

Noise-induced stop-and-go dynamics

- Stop-and-go dynamics in deterministic models results from **inertia, nonlinear dynamics, linear instability phenomena (metastability), and phase transition**
  - Linear instability compensated by nonlinear mechanisms: sensitivity to non-linearity
  - Not generic: Require fine-tuning of the parameters

- **Stochastic cellular automata models** have shown in the 1990's that **noise effects can initiate stop-and-go dynamics**
  - K Nagel & M Schreckenberg. A cellular automaton model for freeway traffic. *J Phys I* 2:2221, 1992
  - R Barlovic, A Schadschneider & M Schreckenberg. Metastable states in cellular automata for Traffic Flow. *Eur Phys J B* 5:793, 1998



NaSch *J Phys I* 2:2221, 1992

- **Noise kick the system out of the steady state** by making it **stochastic periodic** (e.g. oscillating autocorrelation functions)

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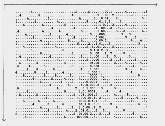

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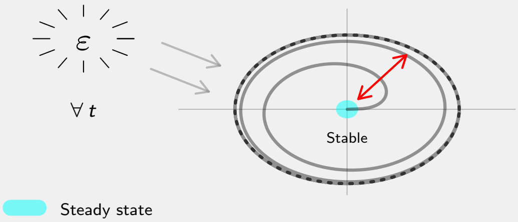
Noise-induced stop-and-go dynamics

Noise-induced oscillation

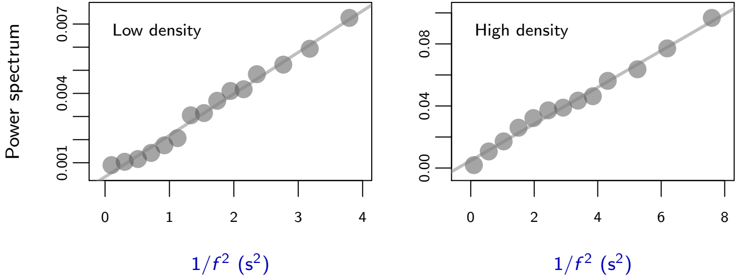
Damped oscillation continuously maintained
by the perturbations



Outline

Nature of the noise in pedestrian speed sequences

Data: ped.fz-juelich.de/database



- **Linear power spectrum** (ACF Fourier transform) of pedestrian speed according to the inverse of squared frequency
- Characteristic of a **Brownian noise** (red noise) to the pedestrian speed

Model based on the Ornstein-Uhlenbeck process

A Tordeux, A Schadschneider (2016) White and relaxed noises in optimal velocity models for pedestrian flow with stop-and-go waves. *J Phys A*, 49(18):1851

- Linear model with **additive noise given by the Ornstein-Uhlenbeck process**

$$\begin{cases} \mathrm{d}x_k(t) &= \lambda\big(x_{k+1}(t) - x_k(t) - \ell\big)\mathrm{d}t + \varepsilon_k(t)\mathrm{d}t \\ \mathrm{d}\varepsilon_k(t) &= -\beta\varepsilon_k(t)\mathrm{d}t + \sigma\mathrm{d}W_k(t) \end{cases}$$

with $(W_k(t))_k$ independent Wiener processes (Brownian motions)

- **Parameters**

Inverse of time gap	λ	$1\ \mathrm{s}^{-1}$	OV function
Pedestrian size	ℓ	0.3 m	
Noise amplitude	σ	$0.09\ \mathrm{ms}^{-3/2}$	Noise
Noise relaxation	β	$0.2\ \mathrm{s}^{-1}$	

- Estimates of the noise relaxation time $1/\beta \approx 5$ s is different from the relaxation or reaction time $\tau \approx 0.5$ s of deterministic models

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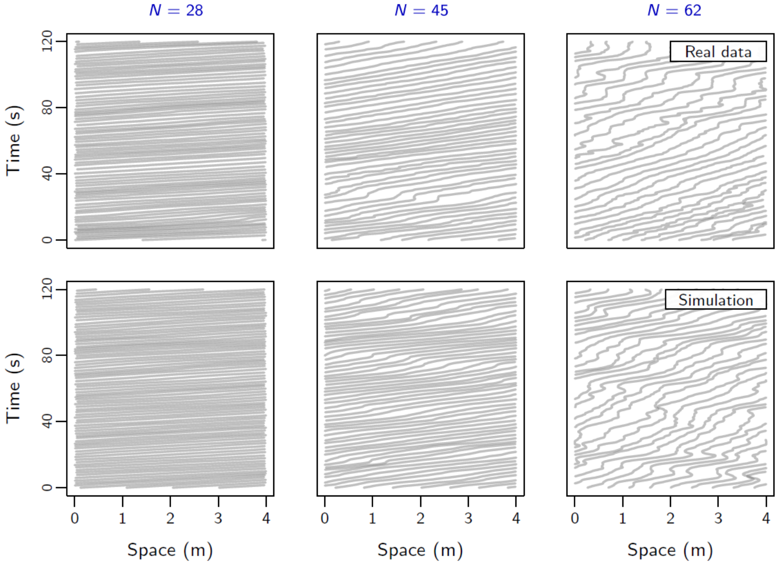
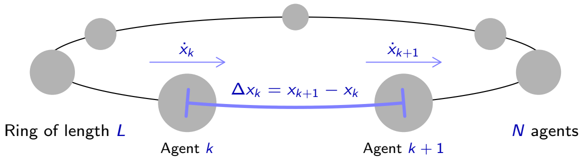


Invariant distribution of the system on a torus

M Friesen et al. (2021) Spontaneous wave formation in stochastic self-driven particle systems. *SIAM J Appl Math*, 81(3), 853-870



- Differential form for the differences $y_k = \Delta x_k - L/N$ to the homogeneous solution

$$\mathrm{d}Y(t) = \big(\lambda A Y(t) + A\Xi(t)\big)\mathrm{d}t, \qquad A = \begin{bmatrix} -1 & 1 & & \\ & \ddots & \ddots & \\ 1 & & -1 & 1 \end{bmatrix}$$

- $Z := (Y, \Xi)$ is a **Markov process** in $\mathbb{R}^N \times \mathbb{R}^N$ (Ornstein-Uhlenbeck and Feller process)

$$\mathrm{d}Z(t) = BZ(t)\mathrm{d}t + G\mathrm{d}W(t), \quad Z(0) = z_0, \quad B = \begin{pmatrix} \lambda A & A \\ 0 & -\beta 1_N \end{pmatrix}, \quad G = \begin{pmatrix} 0 & 0 \\ 0 & \sigma 1_N \end{pmatrix}$$

with generator $Lf(z) = \sum_{k=1}^{2N} (Bz)_j \frac{\partial f(z)}{\partial z_j} + \frac{1}{2} \sum_{k,j=1}^{2N} (GG^\top)_{kj} \frac{\partial^2 f(z)}{\partial z_k \partial z_j}$ Sato, 1984

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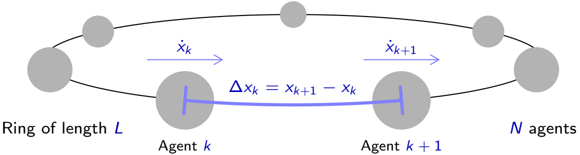

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Theorem *It holds $Z(t) \to Z(\infty)$ as $t \to \infty$ in law, where $Z(\infty)$ is a Gaussian random variable on $\mathbb{R}^{2N}$ with mean zero and covariance matrix*

$$\Sigma(\infty) = \int_0^\infty e^{tB} GG^\top e^{tB^\top} \mathrm{d}t$$

Correlation and autocorrelation

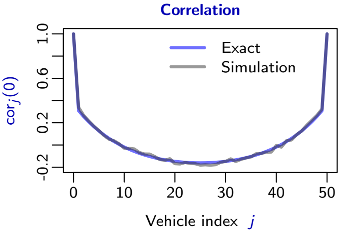
M Friesen et al. (2021) Spontaneous wave formation in stochastic self-driven particle systems. *SIAM J Appl Math*, 81(3), 853-870

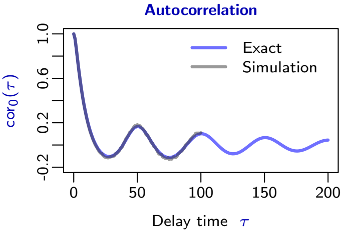
- **Asymptotic covariance of the spacing** of an agent k to the spacing of the agent $k+j$, with $\gamma_n = \exp(i2\pi n/N)$

$$\text{cov}_j(0) = \frac{\sigma^2}{2\beta N} \sum_{n=1}^{N-1} \frac{\gamma_n^j}{\lambda - \beta - \lambda\gamma_n} \left(\frac{(1-\gamma_n)^2}{\lambda - (\lambda+\beta)\gamma_n} - \frac{2\beta}{\lambda(\lambda+\beta-\lambda\gamma_n)} \right)$$

- **Asymptotic autocovariance** at time $\tau \geq 0$

$$\text{cov}_0(\tau) = \frac{\sigma^2}{2\beta N} \sum_{n=1}^{N-1} \frac{1}{\lambda - \beta - \lambda\gamma_n} \left(\frac{e^{-\beta\tau}(1-\gamma_n)^2}{\lambda - (\lambda+\beta)\gamma_n} - \frac{2\beta e^{-\lambda(1-\gamma_n)\tau}}{\lambda(\lambda+\beta-\lambda\gamma_n)} \right)$$





Correlation and autocorrelation at the limit $N, L \to \infty$

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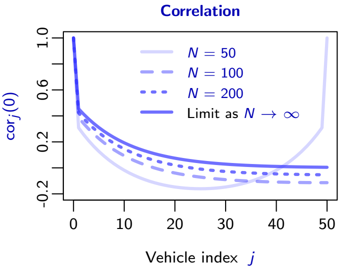
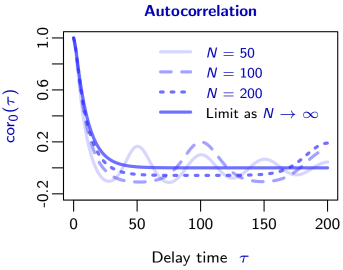
- **Asymptotic correlation of the spacing** in stationary state for the hydrodynamical limit $N, L \to \infty$ with L/N constant

$$\mathrm{cor}_j^\infty(0) = \frac{1}{2}\left(\frac{\lambda}{\lambda+\beta}\right)^j$$

- **Asymptotic autocorrelation** at time $\tau \geq 0$:

$$\mathrm{cor}_0^\infty(\tau) = \frac{\lambda e^{-\beta\tau} - \beta e^{-\lambda\tau}}{\lambda - \beta}$$





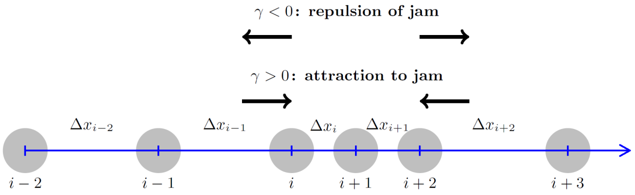
Outline

Extended model with discrete gradient in space

- The noise in the Ornstein-Uhlenbeck model is independent from the dynamics. In addition, the model is unconditionally stable.

- **Extended model:** Coupling of the noise to the spacing difference with the predecessor (discrete gradient in space)

$$\begin{cases} \mathrm{d}x_k(t) = \lambda\big(x_{k+1}(t) - x_k(t) - \ell\big)\mathrm{d}t + \varepsilon_k(t)\mathrm{d}t \\ \mathrm{d}\varepsilon_k(t) = -\gamma(\Delta x_{k+1}(t) - \Delta x_k(t)) - \beta\varepsilon_k(t)\mathrm{d}t + \sigma\mathrm{d}W_k(t) \end{cases}$$



Stability analysis

- Assume $\beta, \lambda > 0$, the **exact string stability condition** is

$$2\gamma\left[\lambda(\lambda+\beta)(1-c_l)^2-\beta^2 c_l\right]+\beta\lambda\left[2\lambda(1-c_l)(\lambda+\beta)+\beta^2\right] - 4\gamma(1-c_l^2)\left[\gamma(1-c_l)+\beta\lambda\right]>0 \qquad \forall l=1,\dots,\lceil N/2\rceil,\quad c_l=\cos(2\pi l/N)$$

  - In the thermodynamic limit for which $N, L \to \infty$ with N/L constant, the condition for the longest wavelength $l = 1$ is

$$\beta\lambda - 2\gamma > 0$$

  - The condition for the shortest wavelength $l = N/2$ is

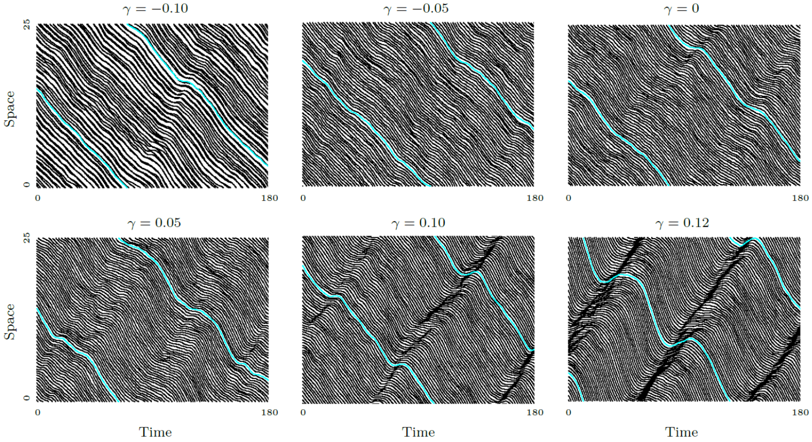
$$\beta\lambda + 2\gamma > 0$$

- **Sufficient linear stability condition:** $\beta, \lambda > 0$ and $-\beta\lambda < 2\gamma < \beta\lambda$

The condition systematically holds for $\gamma = 0$ (initial OU model).

Examples of trajectories

Simulation results with 50 agents, $T = 1$, $\beta = 0.2$, $\sigma = 0.05$ $|\gamma^\star| \approx 0.1283$



Autocorrelation of the speed

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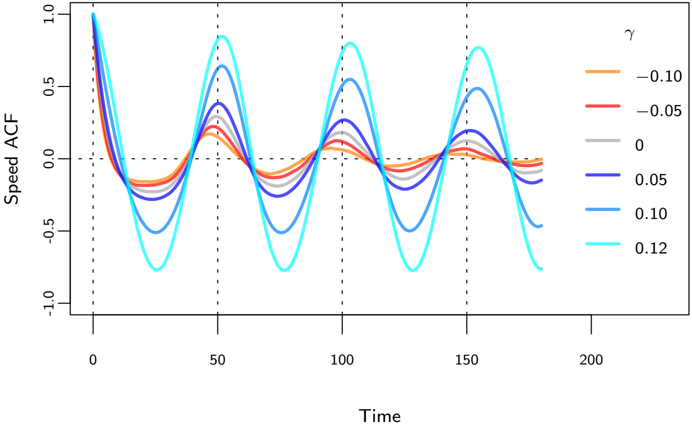

Outline

Nonlinear car-following models

- Both linear stochastic models based on the Ornstein-Uhlenbeck process can describe qualitatively stop-and-go waves observed in pedestrians dynamics
  - Evanescent waves for the unconditionally stable OU model
  - Subcritical instability for the OU model including a gradient in space
- These stochastic models are linear and ergodic[1]: they do not recapture the phase transition observed in vehicular dynamics
  - White noise cannot influence the stability properties of linear models

- **What about nonlinear car-following models?**

  Can the stability properties and long-term behavior of nonlinear models be impacted by white noise?

  → Yes! c.f. Kapitza inverted pendulum

[1] They have a unique and stable (normal) stationary distribution

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Adaptive time gap model

Overdamped local and stable for any $T, \tau > 0$

- The time gap is the distance gap divided by the speed

$$T_k(t) = [\Delta_k(t) - \ell]/v_k(t) = g_k(t)/v_k(t), \qquad v_k(t) = \mathrm{d}x_k(t)/\mathrm{d}t.$$

$\rightarrow$ The time it takes to collide at constant speed if the predecessor stops.

- Assume that the time gap $T_k(t)$ is relaxed to a desired time gap parameter T

$$\mathrm{d}T_k(t) = \frac{1}{\tau}\big[T - T_k(t)\big]\,\mathrm{d}t, \quad T \approx 1.2 \text{ s}, \quad \tau \approx 5 \text{ s}$$

- Newtonian formulation: $\mathrm{d}v_k = \frac{1}{T_k}\big[\lambda\,(g_k - Tv_k) + \Delta v_k\big]\,\mathrm{d}t$

- Stochastic formulation: $\mathrm{d}v_k = \frac{\lambda\,(g_k - Tv_k) + \Delta v_k}{T_\varepsilon(\Delta x_k, v_k)}\,\mathrm{d}t + \sigma \mathrm{d}W_n$

where T_ε is a bounded molifier of the time gap and W_k are independent standard Brownian motions with volatility σ.

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- Stochastic formulation: $$\mathrm{d}v_k = \frac{\lambda\,(g_k - Tv_k) + \Delta v_k}{T_\varepsilon\big(\Delta x_k, v_k\big)}\,\mathrm{d}t + \sigma \mathrm{d}W_n$$

where T_ε is a bounded molifier of the time gap and W_k are independent standard Brownian motions with volatility σ.

Phase transition as the noise volatility increases

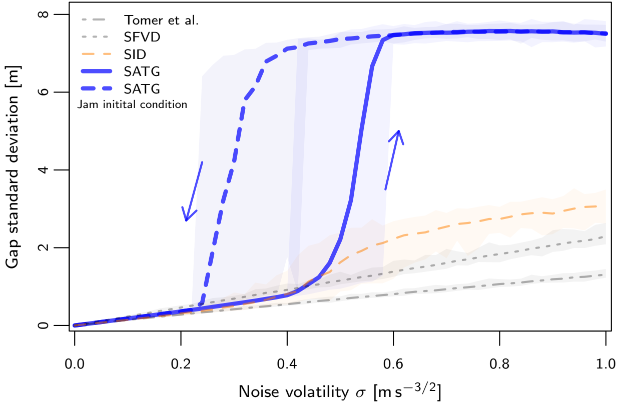


Figure: The curves are Monte Carlo averages, while the areas show the min/max range.

Phase diagram

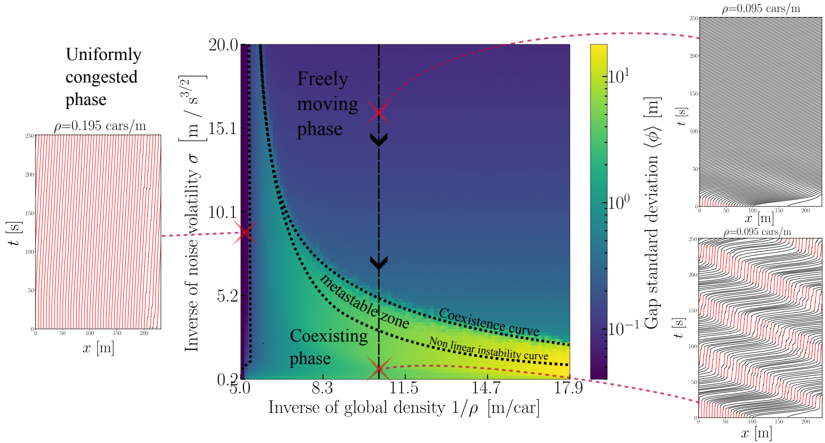
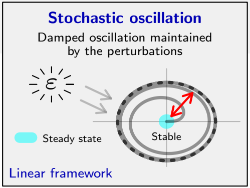


Figure: Phase diagram of the stochastic ATG model in the $(L/N, 1/\sigma)$-space.

Outline

Noise-induced stop-and-go

Summary



- **Stochastic oscillation:** Oscillation of the system at its own deterministic frequency (longest wavelength) due to the stochastic perturbations

  → Evanescent (unstable) stop-and-go waves with **no phase transition** (linear ergodic framework with unique stationary distribution)

  → Subcritical instability: wave amplification near the critical linear instability setting

- **Phase transition:** Nonlinear models with unstable uniform equilibrium solution

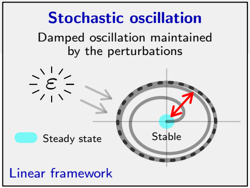
  → Delay-induced (classic): Linear instability – Stop-and-go for fine tuning of the parameters: linearly unstable dynamics hard to control

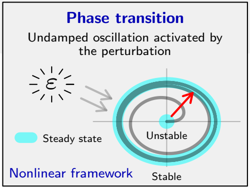
  → Noise-induced: Nonlinear instability for large perturbations – Unexpected results: the deterministic model is unconditionally deterministically stable

- **Linear stability not sufficient to control stop-and-go dynamics in stochastic systems**

Noise-induced stop-and-go

Summary





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Many thanks for your kind attention!

Division for traffic safety and reliability
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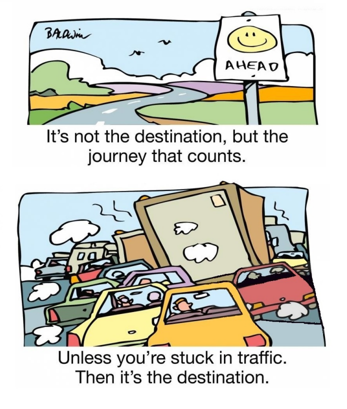

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