Characteristics in stationary state of a Markovian jump process modelling traffic flows

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\section{Objective}
We calculate some macroscopic performances, like the flow-density diagram, of an uni-dimensional continuous time traffic flow model by using a microscopic stochastic jump process and analytical probabilistic tools [1-2].

\section{Presentation of the model}
\begin{itemize}
\item An infinite one-way lane is divided in identical cells of length $\delta$.
\item A zero-range process describes the evolution of the distance gap of the vehicles [3].
\item In the model, we suppose that the speed of each vehicle is a function $g$ of the distance gap.
\item For a vehicle $x$, the jump rate $b$ depends of the number $\eta(x)$ of free cells in front:
\[
 b_0(\eta(x)) = \frac{1}{\delta} \times g(\eta(x)) \times \delta.
\]
The process $(\eta(x), x \in \mathbb{Z}, t \geq 0)$ is a Markov jump process defined by the generator $L f(\eta) = \sum b_0(\eta(x)) [f(\eta') - f(\eta)]$ where $\eta'(z) = \eta(z) \times \mathbf{1}_{x \neq x-1}, \eta'(x) = \eta(x-1), \eta'(x-1) = \eta(x-1)+1$
\end{itemize}

\section{Study of the stationary state}
\begin{itemize}
\item Stationary distribution. The invariant distributions $\pi_z$ of the number of free cells in front are known [4]:
\[
 C_0 \cdot \pi_z(m) = \begin{cases} 
 z^n \prod_{i=1}^m b_0(n)^{-1} & 0 \leq m < D/\delta \\
 z^n \prod_{i=1}^{D/\delta} b_0(n)^{-1} & m \geq D/\delta
\end{cases}
\]
with $C_0 = \sum_{m=0}^{D/\delta} z^n \prod_{i=1}^m b_0(n)^{-1}$
\item Macroscopic quantities. The distributions $\pi_z$, linked to the mean speed $V$, allow to calculate the :
\begin{itemize}
\item mean distance gap $D_l$
\item mean distance gap $D_l(V) = \frac{1}{\Delta[V]} \left( \sum_{m=0}^{D/\delta} m \prod_{i=1}^m b_0(n)^{-1} \frac{D/\delta - 1 - \sum_{m=1}^D c^{D/\delta - 1} \prod_{i=1}^m b_0(n)^{-1}}{\sum_{m=0}^{D/\delta} c^{D/\delta - 1} \prod_{i=1}^m b_0(n)^{-1}} \right)
\item density $g(V) = \frac{1}{\Delta[V]}$
\item flow $Q_l(V) = \frac{V}{\Delta[V]}$
\end{itemize}
\end{itemize}

\section{Study of the case $\delta \to 0$}
When the space scale $\delta$ tends toward zero, by inverting the variables of density $\rho$ and mean speed $V$ :
\[
 V(\rho) \to V(\rho) = \min \{ \rho, g(1/\rho - \ell) \} \quad Q_l(\rho) \to Q_l(\rho) = \rho \times \min \{ \rho, g(1/\rho - \ell) \}
\]
In fact, the model tends toward a deterministic one in which the vehicles repartition is homogeneous.

\section{Examples of flow-density diagram}

\section{Introduction of a reaction time $T^r > 0$}
At time $t$, the jump rate for a vehicle $x$ is $b_0(\delta \times \eta_{x-T^r}(x))$ where the delayed distance gap $\delta \times \eta_{x-T^r}(x)$ is approximated by :
\[
 \delta \times \eta_{x-T^r}(x) \approx \delta \eta(x) + T^r (g(\delta \eta(x)) - g(\delta \eta(x+1)))
\]
The jump rate depends on the states of the considered vehicles and its predecessor. The process is defined by the generator $L f(\eta) = \sum b_0(\eta(x), \eta(x+1))[f(\eta') - f(\eta)]$ where $\eta'(z) = \eta(z) \times \mathbf{1}_{x \neq x-1}, \eta'(x) = \eta(x-1), \eta'(x-1) = \eta(x-1)+1$

\section{Conclusion}
The reaction time generates some kinematic wave like in the car-following models by differential equations.

\section{References}

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